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Semantic Foundation of Diagrammatic Modelling Languages

Applying the Pictorial Turn to Conceptual Modelling

Diplomarbeit

im Studienfach Informatik
vorgelegt von

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Abstract

The following thesis investigates the applicability of the pictorial turn to diagrammatic conceptual modelling languages. At its heart lies the question how the "semantic gap" between the formal semantics of diagrams and the meaning as intended by the modelling engineer can be bridged. To this end, a pragmatic approach to the domain of diagrams will be followed, starting from pictures as the more general notion.

The thesis consists of three parts:
In part I, a basic model of cognition will be proposed that is based on the idea of conceptual spaces. Moreover, the most central notions of semiotics and semantics as required for the later investigation and formalization of conceptual modelling will be introduced. This will allow for the formalization of pictures as semiotic entities that have a strong cognitive foundation.

Part II will try to approach diagrams with the help of a novel game-based FCA technique. A prototypical modelling attempt will reveal basic shortcomings regarding the underlying formal foundation. It will even become clear that these problems are common to all current conceptualizations of the diagram domain. To circumvent these difficulties, a simple axiomatic model will be proposed that allows to link the findings of part I on conceptual modelling and formal languages with the newly developed concept of «abstract logical diagrams». As an outlook, the outline of a categorical model that adjusts the basic lack of a rigorous foundation will be presented.

In the final part III, the discussion of conceptual graphs (CG) as an example of diagrammatic conceptual modelling languages will demonstrate a pragmatic, hands-on approach to the semantic gap. Several alternative semantic foundations of conceptual graphs will be compared, and their relation to the basic ideas of diagrammatic semantics will be elaborated. The thesis concludes with a practical modelling example that reveals the limits of this graphical formalism. After a detailed metaphysical and formal ontological meta-analysis of a simple domain, an extension of CG, in the form of conceptual graphs with relators, is proposed that allows to model the example.

Nimic nu se explică,
nimic nu se dovedeşte,
totul *se vede*.

Emil Cioran
[Cioran 2001, p116]

Nothing is explicated,
nothing is derived,
everything *can be seen*.

Preface

Over the recent years, the computer science community has become aware of pictorial representations as an important formalism and tool for everyday work.

A first indication is the upsurge of diagrams in software engineering and the omnipresence of UML. (There are even attempts to directly translate UML into executable code.) But why do software engineers prefer a graphic notion for object-oriented analysis and design?

Even before the now ubiquitous semantic technologies entered the focus of software engineering, knowledge bases have been an integral part of large applications and theoretical research. In the majority of cases, the interface between a human user and the knowledge base has been a formal representation language like FOL or KIF. Today, knowledge bases have become reduced to an everyday item for the normal computer user. The simple question arises: How to avoid these formal languages from the stone age of computing, such that an average person, in his role as a domain expert that completes his personal knowledge acquisition task, can manage a knowledge base without a PhD in logic?

Sketches are the real lingua franca of science. They are used to visualize results in publications, explain novel outcomes to fellow researchers or students, and even play an important role in the creative act of drawing conclusions which lead to new results. The question, how to deal with these notions of diagrammatic creativity, especially regarding their opposition to the classical way of publicating in linear text, is still open. Further, an implementation of these creative processes in (software) tools for the scientist or engineer is still far ahead.

These first, rather naïve, and hyperbolic approaches to the domain of diagrammatic representation reveal most of the basic demands which antecede this thesis. a practical implementation of diagrammatic techniques require a prior in-depth understanding of the basic research object, which can be circumscribed as: the application of diagrams in the creative acts of humans and the importance of these diagrams as externalizations of internal mental models.

Regarding the following mainly theoretical approach, creative acts will be restricted to conceptual modelling and the entire discussion of the interplay between mental representations and external objects will be limited to the perception of concepts.

Ontogenesis of this Research

This thesis can be seen as the agglomerate of a variety of research topics that I encountered during my course of studies. First and foremost, it combines the mathematical rigour of (theoretical) computer science with the extensiveness of philosophical approaches (following Deleuze, philosophy as "the art of forming, inventing, and fabricating concepts"). Conversely, computer science heavily depends on results, ideas, and conceptualizations from other research areas; hence, doing research in computer science is inherently inter-, trans-, and multidisciplinary.

This work originated from two fields of research: the ideas of Bildwissenschaft which I encountered as a member of the Scientific Visualization project in Halle/S., and the foundation of conceptual modelling languages with the help of formal ontologies which formed part of my research in the Onto-Med group.

I myself prefer sketches and diagrams as a way of approaching scientific research and (software) engineering as well as sharing ideas and explicating the crux of a matter – I am "thinking visually" and my creativity heavily depends on visual representations be it as an internal mental model or as an externalized diagram.

The combination of these three pillars explains my wild enthusiasm for drawing interconnections between a theoretical approach towards pictures, and classical questions of computer science and logic. Further, diagrams will not appear only as research objects, but constitute a central paradigm of the following research methodology.

Acknowledgements

Writing and science in general is – as is commonly known – a constant switching between eremitic thinking and the discussion of results (not to forget the proofreading). I am indebted to all the people who supported this lifestyle.

Above all I will thank the Research Group Onto-Med in Leipzig for supporting my thirst for knowledge in the last semesters of my studies. First and foremost, I am indebted to Heinrich Herre who offered me the chance to write this thesis and supported my investigations beyond the borders of classical computer science with encouragement, help, ideas, criticism, and advice. As representative of my fellow researchers, I will thank Frank Loebe for sharing his expertise in GFO, especially in relations and roles, as well as his support for the odds and ends of writing a thesis.

Further, I am much obliged to Roland Strauß who introduced me to the importance of pictorial presentations and allowed me to make my first research experiences in the Scientific Visualization Project.

And naturally, I am rejoicing at having a host of proofreaders, critical listeners to new, abstruse ideas, and people that kept my non-thesis activities on life support – or, in a single word, “friends”. Thanks, Dominic, Thorsten, Angelika, Katja, Karin, Kai, Teresa, Christine, Irmi, . . . !

“*The most I can do for a friend is simply be his friend*” [H. D. T.] – nevertheless, thank you, Silvio, for your continuous long-distance support and your motivating stimuli especially in the final phase of finishing this thesis.

And, both obligatory and frankly, I am deeply indebted to my family for their support and for keeping me grounded.

This version of the thesis is indentical to the Diplomarbeit that was submitted in mid August 2007 apart from minor typographic changes and corrections.

version tag: 20070820

Contents

List of Figures

0 Introduction

Before presenting an outline of this work, the underlying motivation will be introduced with its anchor in the scientific tradition of Bildwissenschaft. Finally, the interdependencies of this work's sections will be depicted, thus a "map" of this thesis is given to the reader who, additionally, will get basic guidance on the typographic conventions that will be used in the following.

0.1 Motivation: The Pictorial Turn in Computer Science

The maxim of the *pictorial turn* – a term coined by W. J. T. Mitchell [Mitchell 1994] – forces a shift of advertence towards pictorial presentations, against the "*general anxiety [. . .] about visual representation*" [Mitchell 1994, p12][1]. As Martin Schulz notes in [Schulz 2005], the pictorial turn is not merely a continuation of the linguistic turn[2], but, contrariwise, a new approach that starts from a pragmatic, practical perspective towards one of the most basic cultural achievements: pictures and diagrams.

pictorial turn

Background

Pictures and diagrams play an important role in the creative process of understanding. They have been part of Human's intellectual apprehension of their surrounding world since the first attempts to utilize conceptualization for this task or – with Deleuze [Deleuze & Guattari 1990] – when practising philosophy in its most basic sense: "*philosophy is the art of forming, inventing, and fabricating concepts*".

Concepts like εικόν (Greek: 'eikon') and imago (Latin), which referred to the most basal results of perception, are still present in today's terms "icon" and "image"[3]. Nevertheless, the ideas behind these concepts have only been investigated marginally for the last centuries that were dominated by adopting linear language as a central paradigm of epistemology and as the main method in scientific modelling.

The last few years, an interdisciplinary approach brought pictorial representations back into the focus of research and resulted in new branches as "Bildwissenschaft"[4], which could be seen as an agglomerate of disciplines ranging from semiotics and history of art, over computer science and psychology, to philosophy (see [Sachs-Hombach 2005] for a collection of essays from authors of these different areas).

[1] the notions of "imagic turn" [Fellmann 1991], "iconic turn" [Boehm 1994] and "visualistic turn" [Sachs-Hombach 2006] are alternative concepts which, in the following, will be regarded as equivalent to Mitchell's pictorial turn;

[2] the «linguistic turn» subsumes many different paradigms, but can be seen as the major step of mid 20th century philosophy [wp:linguistic turn] [Rorty 1967];

[3] a history of these terms and their different linguistic usage is given in [Scholz 1991]; these differences form a starting point for later investigations;

[4] to avoid homonymy of renderings of this original German term into English (as "picture science" or "science of pictures") with already occupied termini technici, the German title will be used within this thesis;

These different approaches do not only vary with respect to their scientific background, but also in their way of formalizing the basic definitions and results.

Pictures & Diagrams in Computer Science

This work focuses on the impact of the pictorial turn on computer science, especially its implications for conceptual modelling and formal languages.

To this day, pictures have played only an underpart in computer science: on the one hand, they are used extensively in practical conceptual modelling, e.g., software engineering's UML diagrams, and in the areas of image recognition, on the other hand, they are not integrated adequately in the general research on knowledge representation which still clings to sentential languages[5] such as logic.

[5] the language's attribute of "linear" or "sentential" is opposed to "diagrammatic" that – without regarding the distinctions of ch. 7 – equals "pictorial";

Furthermore, pictures are central to topics like graphic software applications, image databases, image recognition, human-computer interfaces, and new research areas like diagrammatic reasoning and representation [Anderson et al. 2002]. Again, all of these research fields are approached in the "classical" way by techniques that originate in sentential logic, e.g., [Süttenbach 2001]. In contrast, the pictorial turn would propose accessing this domain from the side of the visual phenomena and the pragmatic usage of pictures.

0.2 Research Questions

This thesis will focus on practical conceptual modelling with diagrammatic languages and explore the underlying semantic foundation which plays the central role in the modelling. Regarding the pragmatic focus of the pictorial turn, the semantics of pictorial presentations that is inherently based on visual perception will play an important role besides the formal semantics of a knowledge representation language. The following research question restates the prominent role of the underlying paradigm.

Main Research Question

> What are the results of applying the idea of the pictorial turn to diagrammatic conceptual modelling languages, especially regarding their semantic and ontological foundation?

explorative question

Obviously, the nature of the previous research question is at heart explorative; thus, the following analysis cannot lead to a positive or negative result but will provide first steps in a novel area of research. Furthermore, the abstractness of the underlying domain does not allow for empirical research either – at least at the following elementary steps.

subsequent research questions

Above all, the following concepts need to be analyzed prior to approaching the above main research question: «conceptual modelling», «semantic and ontological foundation», «the diagrammatic, i.e., diagrams». The investigation of modelling languages will lead to «formal languages» in general and their relation to the underlying formal semantics in contrast to the meaning as intended by the modelling engineer – the «semantic gap». Semantics in the context of pictures will entail a cognitive view of pictorial «perception» which will constitute the picture's importance over classical, linear language approaches.

0.3 Outline

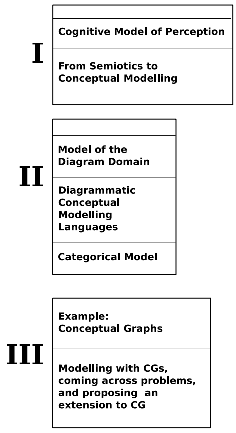
This thesis is subdivided into three main parts that mirror the basic analytical approach.

Part I will introduce the fundament of the following analysis. As pictures are both semiotic entities and objects of perception, a basic understanding of these two research areas is inevitable. First, a basic model of cognition will be proposed that is based on the idea of conceptual spaces. Second, the most central notions of semiotics and semantics as required for the later investigation and formalization of conceptual modelling will be introduced.

Part II will try to approach diagrams with the help of a novel, game-based FCA modelling technique. This attempt requires a prior summary of the most important scientific results regarding pictures and diagrams. Hence, contemporary theories and discussions will be outlined and interrelated regarding a meta-model that underlies Bildwissenschaft.

A prototypical modelling attempt will reveal basic shortcomings regarding the underlying formal foundation. It will even become clear that these problems are common to all current conceptualizations of the diagram domain. To circumvent these difficulties, a simple axiomatic model will be proposed that allows to link the findings of part I on conceptual modelling and formal languages with the newly developed concept of «abstract logical diagrams». As an outlook, the outline of a categorical model that adjusts the basic lack of a rigorous foundation will be presented.

In the final part III, the discussion of conceptual graphs as an example of diagrammatic conceptual modelling languages will demonstrate a pragmatic approach to the semantic gap. After an introduction to conceptual graphs, their origin, and the surrounding formal language framework, several alternative semantic foundations will be compared, and their relation to the basic ideas of diagrammatic semantics (as explicated in part II) will be elaborated.



(due to fig. A.3)

A practical modelling example will reveal the limits of this graphical formalism. After a detailed metaphysical and formal ontological meta-analysis of the example domain's background, an extension of CG, in the form of conceptual graphs with relators, will be proposed that finally allows to model the example.

A more comprehensive overview of the structure of this thesis is given in fig. A.3 at p159 which additionally includes the relations between the sections and points out the important examples. This diagram could serve as a route map for readers that are not willing to follow the trail which is suggested by the linear order of the following chapters.

As the three parts cover a large area of research topics that are firmly anchored in different scientific areas with different underlying research paradigms, their style of representation differs. The first part includes results of artificial intelligence, cognitive science, linguistics, logic, and philosophy; the second combines these different ideas with the help of Bildwissenschaft and extends the resulting agglomeration with a formal basis via conceptual modelling. The third part starts in the classical computer science way of introducing a formal language but includes results from formal ontology and metaphysics to tie the formal framework to a concrete domain.

Computer science's rigorous demand of a formal foundation for all presented results influences the incorporation of outcomes of other scientific paradigms. The idea of dissecting and describing a phenomenon is formalized with the notion of (conceptual) modelling whose central aim is the extraction of abstract structures by sustaining mathematical rigour. Conceptual modelling demands the usage of a (semi-)formal language and the explicit introduction of the underlying basic assumptions. Consequently, results of other researchers have to be re-formalized to fit into this thesis' discussion.

0.4 Reading Guide

This document utilizes typographic features whose availability eases the reading, and transports the idea of hyperlinks to the printed version. In the following, some of these components will be introduced.

Wikipedia- Wordnet- & Stanford Encyclopedia-Links

As most of the embedded hypertext links of this thesis' first part are to the Wikipedia Encyclopedia, these references are not given in standard HTTP-style but as wiki links which are easier to read. Like normal references, these are resource locators in square brackets which are composed of the name of the wiki, followed by a colon, and the name of the wiki page, e.g., [wp:GFO] refers to the page about GFO in the Wikipedia ('wp'), which can easily be translated into `http://en.wikipedia.org/wiki/` followed by the page name GFO. Since literal citations need to refer to a stable version, some links have an additional date string in angle brackets which corresponds to a permanent link. The PDF-version automatically transforms wiki-links to standard HTTP-URLS even for these permalinks [wp:Permalink].

Wordnet

The same wiki-link style is applied to references to the WORDNET Lexical Database [Fellbaum 1998] where the wiki-name is abbreviated as 'wn' and the URL is based on the online electronic version `http://wordnet.princeton.edu`. Analogously, articles of the Stanford Encyclopedia of Philosophy [Zalta 2007] are cited by the wiki-name 'sep' and the URL `http://plato.stanford.edu/archives/spr2007/entries/` corresponding to the spring 2007 edition.

Stanford Encyclopedia

Peirce's Works

In accordance with the philosophical literature, Peirce's œuvre is not cited in this work's BIBTEX citation style but in the consuetudinary way as [CP, <number>] for the Collected Papers's article <number> as published in [Peirce 1994].

Hypothesis vs. Postulate vs. Definition

The three layout environments *hypothesis*, *postulate*, and *definition* underline a proposition's dependency on others. Postulates state basic assumptions which are not grounded further, reminiscent of axioms in the axiomatic deductive method as defined at page 46. Definitions are used according to this method: they generally define a term of certain importance for the following work in such a way that only other definitions or postulates are used in the definiens. New terms that are not defined explicitly are emphasized by margin notes.

Hypotheses are statements that form the basis of a subsequent investigation which is either brought forth in the following text or only outlined briefly because of its dependence on further (empirical) study beyond this thesis.

Glossary

The glossary starts in appendix 1 at page 180. Entries are marked at their first introduction and at important occurrences in the succeeding text by a superscript small right arrow ("$^{\rightarrow}$"). Compared to definitions, the glossary only outlines a selection of background concepts and accentuates certain aspects which are important for the progress of this work's central thread of discussion.

Star-Sections

(Sub-)Sections whose titles ends with a star (∗) contain additional information. They show a different perspective on previously discussed matters without going into detail, present ancillary references to literature, or show cross-links for readers familiar with topics beyond this work's scope. Hence, they depend on basic knowledge in advanced topics that was not introduced earlier in the step-by-step manner of this thesis. A double star indicates an expert's view that is beyond the scope of this thesis' level of detail but which allows to reformulate results from different points of view, e.g., GFO or UML versions of given definitions, or to show connections to other, more sophisticated research areas.

Citation of Figures

Figures and diagrams are often 'cited' in an enhanced style: Instead of simply including the figures one-to-one, they are either simplified, assembled into one, or some graphical elements are changed to embed the original icons into this thesis's iconic language (see appendix H). Contrary to textual citation rules, there is no simple and easy way to annotate those changes in situ; therefore, they are indicated by a "due to" in the figure's caption.

References to figures that were included previously are accompanied by a small pictorial citation on the margin which allows to reminisce the pictorial representation without jumping back to the original page in the document.

Mathematical Shorthand

Although this thesis' languages tries to avoid mathematical shorthand, their conciseness and brevity are nevertheless useful when stating definitions. In order to avoid confusion for the unaccustomed reader as well as for the connoisseur, the most commonly used abbreviations will be presented:

iff	if and only if, i.e, it is necessary and sufficient
f.e.	for each of the following elements
s.t.	such that the following condition holds
: (colon)	emphasizes the definitory character of the following symbol, e.g., "definiens :iff definiendum"

Symbolic Language

This thesis employs an original set of icons as part of a symbolic language which is applied in diagrams and as part of their accompanying text[6]. An overview of the icons used can be found in appendix H.

[6] this feature will be addressed in sect. 11 as heterogeneity;

Part I

Fundamental Notions:

Cognition – Semiotics – Semantics

Approaching Pictures via Cognition and Semiotics

The following chapters will introduce this thesis's theoretical fundament starting from the anticipation of next part's basic postulate about the nature of pictorial representations (see p60).

Main Axiom 1

pictures

> Pictures are signs that are close to perception ["wahrnehmungsnah"].
> [Sachs-Hombach 2006, p74] (own transl.)

From here, two different research areas are entered simultaneously: first, pictures considered as signs lead to semiotics and semantics; second, perception is a cognitive act and therefore bridges the precedent symbolic entrée to an underlying neurobiological process in a special, "close" way which causes the uniqueness of the concept «pictures».

As pictures will be (partially) subsumed under signs, most of their characteristics are inherited from the more generic concept. To quote a popular definition, a *sign* is "*...something that stands for something else, to someone in some capacity*" [Danesi & Perron 1999][7]. Consequently, a picture is (a) *something*, e.g., an object, which is used by *someone* (b), e.g., in an act of communication; therefore, (c) it is embedded in some *contextual situation*, and further (d) has a connection to some *other entity* which it represents. Hence, the following semiotic investigation will focus on this triadic relation (a,b,d), the role of communication, and the contextual background knowledge regarding the (semantic) relation between (a) and (d); the cognitive analysis must first couch in terms a basic understanding of reality and the processing of *percepts*, i.e., basic units of perception, into mental representations.

sign

[7] cited due to [wp:Sign(Semiotics) <200608121352>]

percepts

The first chapter will discuss the cognitive basics which, in the next step, will lead to a cognitive linguistics's view onto semantics. The proposed cognitive model will be contrasted to different other formalizations, e.g., image schemata and artificial perceptions which are a notion based on category theory. Seamlessly, the next chapter will introduce a semantic framework on the basis of a simple semiotic theory while encompassing notions of formal languages, (formal) ontology and conceptual modelling.

1 Cognitive Basics

Cognition refers to mental representations and mental processes like memory, perception, problem-solving, and mental imagery [wp:Cognition]. As pictures are generally approached by visual perception, most of their features originate from this mental process. In the context of this thesis, the difference between cognition and perception[1] represents the two different basic approaches to artificial intelligence which will be presented in detail in the following; nevertheless, later chapters will consider them equal because cognition and perception cannot be defined without the other.

cognition

[1] following [Barsalou 1999], differentiating between cognition and perception is originally based on a theoretical, purely artificial distinction;

Koffka defined *perception* as even more basic than psychological (high-level) processes – as "*the realm of experiences that are not merely 'imagined', 'represented', or 'thought of'*" [Koffka 1922, p532]. Perception involves "*sensation, association, and attention*" [Koffka 1922, p533] or in more modern terms: perception is a sensory action that is guided by extracting patterns and by mapping these patterns to already known ones. This definition will underlie the following approach of modelling visual cognition.

perception

This section will mainly approach (adult)[2] human cognition; nevertheless, most results, especially the presented formalizations, can be transferred either to the artificial cognition of robots and agents, or to other forms of 'biological' cognition.

[2] additionally incorporating aspects of learning would demand several enhancements to the proposed theory;

1.1 Philosophical Preliminaries

As cognition is about the basic distinction between external objects, internal representation and an intermediary bodily connection, some basic assumptions about these are necessary. Most of them seem to be gratuitous from a common-sense / -science point of view but are important for inferring consequences in the following treatment and therefore will be stated explicitly.

Postulate 2

objectivism

There is a realm of objects that exists independently of one's mind.

Postulate 3

embodied intelligence

We (in the sense of: all humans) have senso-motorical access to these objects; mental processes are interactions of an embodied mind with its environment.

Postulate 4

constructivism

We are able to store mental representations of external objects; these internal objects do not necessarily represent the structure of the real external objects but are rather the product of "categorization" or "conceptualization" (which will be examined and defined more exactly in the following); therefore, our own view onto the world, i.e., our structuring of reality via the objects that we perceive, is primarily a construct of our mind.

Definition 1.1

mental models

These internal objects will be called "mental models" (Mм) or "(mental) images"[3].

[3] as the concepts «image» and «picture» are not used synonymously [Scholz 1991], the pictorial state of these mental images is left out of discussion here (cf. the imagery debate of sec. 3.2);

These postulates fix a certain philosophical tenor and therefore heavily influence the following construct of ideas. Due to the axiomatic method of their introduction, for most of these assumptions no justification can be given, whereas some later examples will give a hint about their underlying motivation.

1.2 Cognition in a Nutshell

A first, rather naïve approach, would model vision and cognition in the following way (fig. 1.1): an object of reality is projected via reflection of electromagnetic waves onto the observer's retina; there, biological sensors transform the optical stimuli into neuronal stimuli which are consecutively processed by a neural network. In this first approach, aspects like bias, noise, the discussion of the blind spot and other sensor-characteristics should be left out.

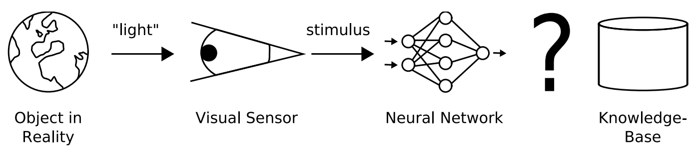


Fig. 1.1:
A first Approach to Vision

Taking into account postulate 4 and its assumption of a high-level internal representation, a model of vision has to cross the *gap* between symbolic representation and neuro-biological activity; this gap remains one of the main problems of cognitive

(neuro-biological) gap

science and epistemology. In order to avoid the question whether mental processes like problem-solving are directly reducible to neuro-biological implementations, a path different from the classical approach will be chosen which will circumvent this discussion.

1.3 Conceptual Spaces

Peter Gärdenfors introduced conceptual spaces in [Gärdenfors 2000] to overcome the differences of today's two antagonistic paradigms of artificial intelligence (AI): on the one hand, the symbolic approach,that defines cognition as (Turing-machine) computation and symbolic manipulation; on the other hand, connectionism and the assumption that mental processes do not work on high-level mental representations. These are merely two ways of approaching cognition; either top-down from symbolic representations and an a priori idea of knowledge close to the everyday usage of language or starting bottom-up talking about sensory stimuli, neural nets, and simple learning algorithms. This dilemma exactly matches the already mentioned gap which is still to be crossed. Conceptual spaces tie those two paradigms together by the question of concept acquisition: language needs a foundation in some sort of basic meaning-bearing units[4]– named concepts, and even a simple task of perception needs some internal representation to be able to *re*-cognize objects. Based on [Gärdenfors 2000], the following definitions are stated:

bottom-up vs. top-down in AI

[4] whether these basic semantic units really exist remains a controversial debate; for the sake of linguistic atomism, i.e., the principle of compositionality (def. 2.9), assuming their existence seems vital at a first, naïve approach;

Definition 1.2

conceptual space

> A *conceptual space* (or concept space) is a linear space→ with an additional quality measure. This *quality* refers to perceivable features of external objects. From this quality an ordering of external objects can be deduced: two external objects seem similar if they are perceived as having the same qualities. This should be called *perception-similar*. Relations among properties are represented by attributes of the space's metric and topology (e.g., being close / far / between).

quality

perception-similar

A stricter mathematical notion would propose a linear space P with an additional mapping $qual : P \to K$ with K an algebra representing the quality[5], e.g., the field of real numbers. (Regarding the linear space of $\mathbb{R} \times \mathbb{R}$, an additional quality dimension $\mathbb{R}$ would be depicted as graph in $\mathbb{R}^3$ whereas the underlying linear space remains 2 dimensional ($\mathbb{R}^2 + \mathbb{R}$).)

[5] the choice of an appropriate algebra is restricted by the demands of following definitions, e.g., topological convexity; Gärdenfors only employs the field $\mathbb{R}$ [Gärdenfors 2000];

Definition 1.3

concept

property

A *property* is a region in a conceptual space. A *concept* relates to certain properties by forming a partitioning of the concept space, i.e., a characteristic function on elements of that geometric space which either belong to the concept or not. Two concepts are *similar* (*concept-similar*) if they share the same properties.

concept-similar

Regarding the mathematical notion, a concept is nothing more than a subspace of the conceptual space whereas the definition of these subspaces must regard both the subspace[6] of the linear space as well as the relation of its elements to the quality measure.

[6] maybe the constraint to linear subspaces is too strong and the notion of subsets of the space's carrier set should be preferred;

Definition 1.4

sub-concept

A concept which can be grounded via its properties directly into perceivable qualities is called *sub-concept*. Therefore these concepts seem to relate more immediately to external objects and are perceivable in a straight-forward manner. This allows to recognize them as a kind of basis for other, more complex concepts[7].

[7] here, basis$^{\rightarrow}$ is also used as in the context of linear spaces$^{\rightarrow}$;

In the course of the following chapter's analysis, also other entities different from the inhabitants of conceptual spaces above will be called "concepts". A fine grained differentiation will be presented after the introduction of all other definitions of «concept»[8] in appendix C (p166).

[8] a concept's name will be given in guillemets, thus «concept» denotes the concept named "concept";

Example 1 : A Simple Conceptual Space

In the pursuit of a simple example of conceptual spaces, the valuation of qualities is to be restricted to binary values (quality is present / is not, $K = \mathbb{B}$). Now, the concept space representations of two external objects will be compared: a globe and a picture of a globe. Fig. 1.2 lists a selection of distinct visual features: a circle, some geometric patterns and a certain way to relate them on the plane, and the existence of a frame around the whole composition.

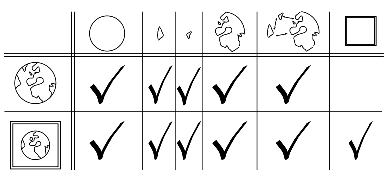


Fig. 1.2: Table of Distinctive (Visual) Features

Hence, we can arrange these features in a binary space. Fig. 1.3 represents the conceptual space (C) and the 'vectors' representing the globe (A) and the picture of it (B).

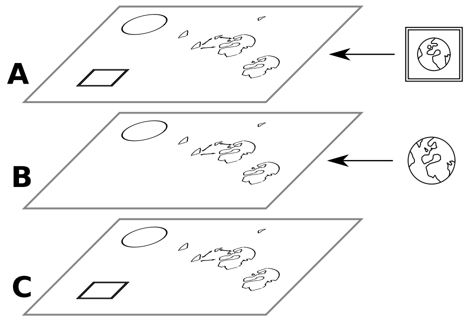


Fig. 1.3:
A Conceptual Space Representation: (C) represents the whole space, (A) and (B) two subspaces

The advantage of conceptual spaces becomes obvious if we extend the binary valuation to allow a fine-grained measure of (perception-)similarity. For example, a frame around the picture does not have to look exactly like the one given above, it can be rectangular or even oval, but it must share some kind of basic perception-similarity. This is expressed by defining a property as a region in the search space; this is depicted by fig. 1.4 which utilizes the above binary properties as prototypes in the class of objects contained in a region of the conceptual space, e.g., a region containing different kinds of frames of pictures.

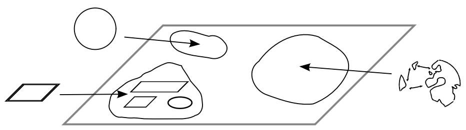
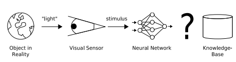


Fig. 1.4:
Properties as Regions

Now, a concept is nothing more than a collection of those regions. To say something belongs to the concept «picture of a globe» is equal to the fact that it bears qualities that belong to certain property-regions; these are tied together by this concept's characteristic function that describes which properties are relevant. In the given example, these properties are represented by the regions 'being framed', 'being enclosed in a circle', and 'having some (continent-like) patterns in a certain arrangement'.

Resuming the actual problem of bridging the gap in the proposed simple model of perception (fig. 1.1), the connection of the neuro-biological connectionist view and the symbolic paradigm is accomplished with the help of conceptual spaces.



(fig. 1.1)

Figure 1.5 gives a first overview of the multi-layer approach which is based on [Chella et al. 1997][9]. This idea's core feature is hidden between the sensory approximation and the formation of sub-concepts: similarity on the sub-concept level is equal to perception-similarity. Therefore, sub-concepts in some way resemble their external origin.

perception similar vs. sub-concepts

[9] as referred to at [Gärdenfors 2000, p251];

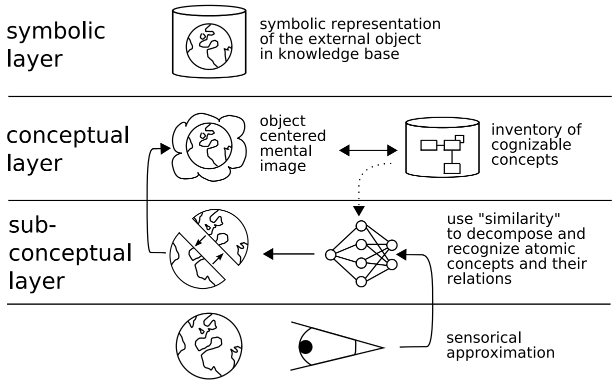


Fig. 1.5: Layered Approach to Cognition (bottom up)

1.4 Matching Percepts and Concepts

There remains the problem of mapping a sensory representation of an external object to a concept. At the layer of conceptual space, this can be described by geometric constructions, e.g., Voronoi or other tessellations as presented in [Gärdenfors 2000, ch. 3 – 4]; nevertheless, these do not serve as a satisfactory solution in the context of the other two layers. Hence, this mapping should first be restated in a bottom-up way. A corresponding top-down view starting from the symbolic layer will be given later in example 15.

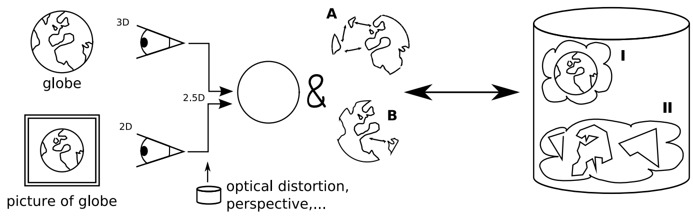


Fig. 1.6: Sub-Conceptual Vision Revisited

Figure 1.6 recapitulates the situation so far. Additionally, the perception of special external objects – pictures, is contrasted to the standard process of vision.

An external object (terrestrial globe, line drawing of it) is mapped to visual properties (a sphere and two possible sub-conceptualizations). There are two possible next steps: either find some (sub-)concept that directly matches the globe, e.g., concept (I), or construct an internal representation on the already existing basic building blocks, e.g., some concepts which represent the already extracted visual properties (recognize known continents (II) on a sphere in some special arrangement) and a way to choose among different possible sub-conceptualizations (continents (A) vs. sea (B)). The construction of the new concept combines already existing concepts in a way homomorphous$^{\rightarrow}$ to the dissection of the original object to percepts. Regarding pictures, their mapping to concepts, especially their dissection by perception and the construction of concepts by homomorphy will be shown to be "close" to the mapping of the real, depicted objects.

1.5 Conceptual Space as Search Space

From an external point of view, a conceptual space is merely a multi-dimensional search space→ . When seeking a conceptual representation of the perceived object, one tries to find a concept with maximal conceptual-equivalence[10]. Therefore, the given model of cognitive vision should be enhanced with a search-algorithm as the final piece of the puzzle. The resulting theoretical account appears in fig. 1.7.

search space

[10] the concept-similarity can be shown to be an equivalence relation;

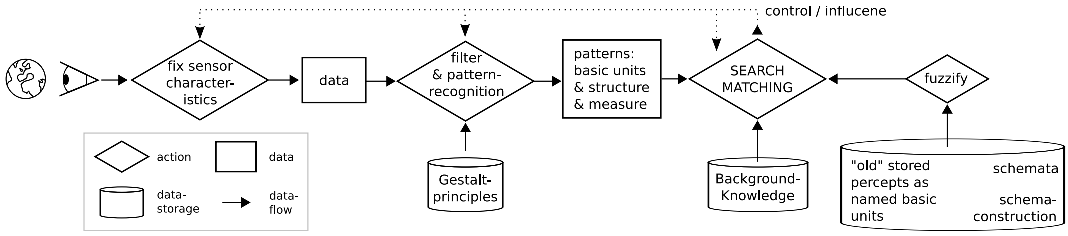


Fig. 1.7:
A detailed View onto Concept Recognition

Additionally, there are some minor enhancements to the generation of the visual properties which form the input to the mapping algorithm. These include the post-processing of the sensor data to smoothen the sensor characteristics or to correct flaws. Typical examples are the human eye's blind spot, colour-vision that is independent of the spectrum of the external light-sources [Foster 2003], or the learnt rules of geometric perspective by which one is able to derive depth vision from two-dimensional drawings (previously mentioned in fig. 1.6)[11]. Furthermore, the extraction of patterns, structure, and basic units is guided by rules which should be abbreviated here as the result of Gestalt effects based on the ideas of Gestalt psychology. These rules describe the formation of patterns from visual data and were first investigated empirically by the Berlin School and its offspring in [Wertheimer 1923] [Koffka 1922] [Arnheim 1988] who claimed that these rules were inherent to all human perception. As with the previously mentioned post-processing, the neuro-biological implementation of these actions is left open, but their postulation is based on empirical evidence, and they allow to metaphorically describe the extraction of patterns as data-flow.

[11] perspective is mostly a culturally trained reading capability [Lopes 1996, p30f] which, nevertheless, is based on basic Gestalt and size perception principles [Sachs-Hombach 2006, p141ff];

Finally, the core of the model is described by a proposed *search algorithm* which implements a pattern matching with additional feedback to the generation of the input data. Therefore, this algorithm combines a filtered version of the visually perceived data, then enhanced by an overlay structure that is defined by Gestalt rules, with a representation of the conceptual search space. The search space is represented by a

search algorithm

storage of previously recognized and memorized concepts and schemata which allow to build high-level concepts from basic sub-concepts; these sub-concepts form a basis of the linear space due to definition 7 and are a compact[12], finite representation which is needed to store the possibly infinite search-space in finite memory.

[12] in the sense of space efficiency, not mathematical compactness of the space;

The important issue is the fuzziness of the pattern matching. Two instantiations of the same concept share the same properties (def. 1.3), but when this restricting equality to properties that are perceivable visually: two concepts (or better: two mappings of external objects to concepts) can exhibit equal visual properties without sharing all their other properties. For example, the property of belonging to the concept «frame of a picture» is based on the visual feature of surrounding a picture whereas the other features, e.g., being wooden, are not important for the mapping to «frame» (but to «wooden frame»).

Definition 1.5

fuzzy-equal

> Two concepts are *fuzzy-equal* if they both share at least their visual properties.

As fuzzy-equality follows from concept-equality, one can introduce and order-relation between all concepts that share a fuzzy-equal kernel. A distance measure in this order can be used to describe the distance between concepts. For example, the concept «wooden frame» is closer to «wooden, rectangular frame» than to «(simple) frame».

The model in fig. 1.7 includes this aspect as fuzzification action, allowing to describe the matching of a prototypical representation of an object (as memorized past perception) to the percept of an object "resembling" the old one. Therefore, these two perception match *up to a certain degree* that would be given by a formal notion of this fuzziness[13]. This leads directly to the next definition.

[13] a formal foundation would include fuzzy sets as well as their statistical underpinning but depend on a prior correct formalization of percepts and the conceptual space;

Definition 1.6

resemblance

> A perception of an external object *resembles* a certain concept if the proposed search algorithm matches these two by fuzzy-equality of a certain degree.

This is no circular definition because resemblance was not part of the pattern matching algorithm but the matching of visual properties in a conceptual search space. With fuzziness and the fact that the continuous[14] conceptual search space can be represented by a storage of previously perceived concepts, sub-concepts, and conceptual combination schemata, the recognition of percepts of previously unknown external objects can be described by resemblance. Thus, the discussion of the improper equation of the meaning of a picture with resemblance, which was laid down in [Goodman

[14] continuous in the following sense: there are infinite possibilities to construct new concepts on top of already known ones;

1968] and will be discussed in the area of pictorial semantics (p76), can be escaped as the lax usage of resemblance becomes constricted to a perceptive basis.

non-deterministic algorithm

Without stating a formal proof, this matching algorithm is non-deterministic because of the possibilities to feed-back onto the basic filtering of the input and the algorithm itself as well as the possible fix point-characterization of the algorithm, i.e., the algorithm recognizes a "good" matching by reaching a loop. Further, its result, by maximizing the fuzzy-equality, can only be a local maximum→ of the conceptual space's quality measure of concept-equality.

background knowledge

After introducing most of the entities participating in fig. 1.7, the role of background knowledge to the search algorithm needs further explication.

Example 2 : Duck-Rabbit

The simple line-drawing in fig. 1.8 has been a source of inspiration to different epistemologists, philosophers, and – especially – psychologists since it's publication 1892 by Joseph Jastrow[15]. In short, one recognizes either a duck or a rabbit – therewith the name "duck-rabbit" – depending on one's pre-assumptions, i.e., the a priori choosing, which visual properties would be favourably filtered out.

[15] for a detailed historical review and references to important publications see [Kihlstrom 2004];

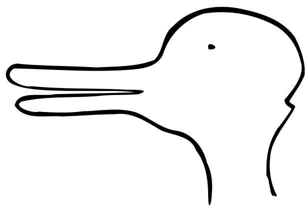
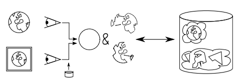


Fig. 1.8:
The famous Duck-Rabbit

Therefore, visual perception depends on pre-assumptions either given by the situation in which perception is accomplished, or by explicit a priori assumptions about the external object. (For example, if the previous duck-rabbit is accompanied by the caption "picture of a rabbit", recognizing the rabbit will be easier.) From an algorithmic point of view, this results in the choice of starting points→ for the search and in constraining the search to certain trajectories→.

1.6 "Closeness" Revisited

On the basis of the proposed model, the perception of pictures can now be contrasted to that of external objects in general. As to be defined later, pictures are two-dimensional, static objects and consequently subject to perception. In retrospect upon fig. 1.6, the patterns extracted from the sensor data of the external object (a 'real' globe) and its picture seem to be similar, or, at least, the same patterns can be



(fig. 1.6)

derived from both. The main difference lies in the post-processing of the sensory data: binocular vision allows stereopsis, i.e., depth-perception of 3 dimensional objects[16]; this results in an augmented 2 D representation (2.5 D)[17]; additional knowledge about representation techniques, e.g., geometric perspective, allows to extract depth information even in diagrammatic (2 D) representations – likewise leading to an internal 2.5 D representation. Subsequently, the search algorithm which works on the derived patterns of the 2.5 D representation is able to derive a matching to the same concept for the perception of a pictorial representation and some other external object that resembles the depicted concept. Additionally, pictorial perception results in recognizing the object *as picture* which introduces extra knowledge to the search algorithm ('being a picture' as kind of super-pattern) and therefore allows to use background knowledge of pictorial literacy→ to influence the perception and mapping process[18].

stereopsis & 2.5 D

[16] leaving out time, as visual perception does not focus this dimension;

[17] here, the dimension 2.5 does not refer to a fractal dimension but to both augmented 2 D and diminished 3 D;

being a picture

[18] chapter 4 will focus on the different point of views on diagrams, especially Küker's three basic ways of approaching a picture (p62) and the layered model of fig. 5.2;

Postulate 1 stated a "closeness" which – in the light of the above discussion – can be summarized as the similarities between the mapping of pictorial representations to concepts and the standard perception procedure of the depicted external objects. Both depend solely on the decomposition of the sensor data into patterns and its mapping to a concept via the composition of sub-concepts. This is different to the perception of a sentential representation like a linear text as this paragraph itself. This sheet of paper is also a 2D, reading static object containing arranged (word-)patterns, though these patterns are not matched to concepts by visual decomposition and structural analysis but by background knowledge's pre-assignment of word patterns to concepts[19]. However, this difference dwindles when using languages like Chinese which allow the composition of ideographic[20] characters, or pictographs of Native Americans whose symbols resemble the depicted concepts and which allow to compose more complicated glyphs from basic ones analogous to the underlying real-world object's composition. The contrast of pictures and language's symbols directly leads to the second import aspect of postulate 1: semiotics (see chapter 2). Before delving into the next linguistic chapter, two additional ways of modelling perceptions and percepts will be introduced which embed the achieved results into the broader field of image schemata and will propose a mathematical formalization which will be adjuvant in part II.

pics vs. text

[19] reading words can be explained as purely symbolic of pictures as icons (p72ff), cf. the ways to approach the Gestalt-semiotic layer in chapter 4, especially the discussion of heterogeneity (p73);

[20] in Chinese, pictographic characters exist, but these are outnumbered by ideographic ones;

1.7 Conceptual Spaces vs. Image Schemata ∗

There is another important theoretical construct that describes the connection of sensomotorical perception and meaning on the symbolic layer. The image schemata of [Johnson 1987] and [Lakoff 1987] play an important role in the area of cognitive linguistics which grounds linguistic features in cognitive operations.

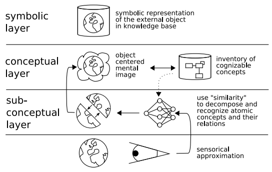
Previous section's concepts are notions originating from the symbolic layer. Bottom-up considerations as in cognitive science would prefer the notion of a recurring image schema.

Definition 1.7

image-schema

"*Image schemata are patterns characterizing invariant structures within topological neural maps for various sensory and motor areas of the brain.*"

[Johnson 2006, p19]



(fig. 1.5)

Figure 1.5 drew a distinction between concepts and sub-concepts; this is analogous to the proposed classification of image schemata into perceptional and prototypical schemata which are the basis for the composition of higher-level schemata [Grady 2006]. To avoid confusion in the following work, image schema and concept should be considered synonymously; whereas the first notion emphasizes the cognitive aspects [Hampe 2006] and the second accentuating the role as foundation for symbol systems.

Postulate 5

Image schemata and concepts are synonymous regarding this work's approach.

Finally, image schemata base the internal representation mechanism more intuitively as part of the mental stratum→ than concept's technical notion. The next axiom therefore grounds mental representations in a natural way to perception which was not possible before:

Postulate 6

mental models

Mental models are image schemata or are built of image schemata plus specific ways of accessing the contained information.

Image schemata allow to include visual perception with other senses and emphasize the importance of the senso-*motorical* feedback-loop between the learning agent and its environment, and, hence, are a more general notion than conceptual spaces. Besides, they cross the "gap" in a way that is closer to a possible neuro-biological implementation than conceptual spaces' abstract algebraic modus operandi. Thus, ideas from Gestalt theory and cognitive linguistics can easily be connected to this notion. On the other hand, the focus on conceptual spaces allows to describe visual perception in an algorithmic way avoiding the scientific discussion of the underlying neuro-biological implementation.

1.8 Modelling Perception with the Help of Category Theory $**$

As will be stated in section 2.5, a conceptual model needs a formalization in a formal language. Category theory allows a highly abstract and formalized notion underpinning the above theory with a formal semantic model[21]. Since a detailed model will not be necessary for the discussions of the remaining chapters, only a basic approach will be sketched here based on the work of Zippora Arzi-Gonczarowski. This section includes references to advanced topics which play important roles in later sections like formal concept analysis (FCA), formal languages, «truth», and logic. Further, a basic familiarity with category theory is presupposed (thus $**$, see sect. 10.1 for references to basic literature). Another approach that bridges the gap between a neural net implementation and graphical representations is presented in [Healy 2000] and [Healy & Caudell 2006], which directly connects a category of neural nets to a simplified category of concepts. Lacking Arzi-Gonczarowski's strong focus on concepts, Healy's approach concentrates on the neuronal implementation of perception.

[21] in the sense of [Burstall & Goguen 1977]: a well-structured description with precise semantic definitions that make clear the inherent structure;

1.8.1 Category of Artificial Perceptions

First a few words on the differences between the approach of sect. 1.5 and Arzi-Gonczarowski's undertaking in both [Arzi-Gonczarowski & Lehmann 1998b] and [Arzi-Gonczarowski & Lehmann 1998a]: she focusses on artificial perception instead of the human centred vision above, and mentions Gärdenfors' conceptual spaces as possible inspiration which is formalized by her approach (without actual proof); instead of proposing a way to cross the gap of cognition, a quality measure of the mapping between objects and percepts becomes vital. This matching is expressed with the help of truth-values of a simple three-valued logic: **t** (true), **f** (false), and **u** (still undefined), which are based on a pragmatic quality measure[22].

[22] cf. the discussion of truth values and their role in sect. 2.3, especially regarding formal semantics, as well as the role of a pragmatic (background) context which pervades the discussions of ch. 2;

Besides these differences, the categorical[23] approach exhibits structural similarities with other approaches presented in this thesis. Before incorporating the ideas in an all-embracing model in chapter 8, a first, basic introduction will be given. Without explicit citation, the following definitions are all based on the two publications [Arzi-Gonczarowski & Lehmann 1998b] and [Arzi-Gonczarowski & Lehmann 1998a].

[23] categorical emphasizes the connection to category theory whereas categorial will be used for categorization as used in the contexts of semiotics and modelling;

Definition 1.8

A *perception machine* (or short: *perception*) is a three-tuple $<\mathcal{E},\mathcal{I},\rho>$ such that $\mathcal{E}$ and $\mathcal{I}$ are finite, disjoint sets and ρ is a three-valued predicate $\rho : \mathcal{E}\times\mathcal{I} \rightarrow \{\mathbf{t}, \mathbf{f}, \mathbf{u}\}$.

perception machine

The elements of $\mathcal{E}$ are objects of the external world whereas the mental images are represented by $\mathcal{I}$, the set of *connotations* which is a collection of entities similar to

connotations

FCA's intensions, i.e., properties. The *perception predicate* relates the mapping of an external object and an internal connotation with truth elements. Thus, reminding of an extended FCA incidence relation (def. C.1) whereas $\rho(w, \alpha)$ equalling either **t** or **f** denotes that the object w either has or lacks the connotation α and **u** that it neither has nor lacks the connotation but will be defined in the future to either one of **t** or **f** [24].

perception predicate

[24] as discussed by Arzi-Gonczarowski, this is a Łukasiewicz style interpretation of the third truth value;

For the following considerations, the real-world $\mathcal{E}$ is held fixed such that perceptions can be written as $\mathcal{P} =< \mathcal{I}, \rho >$. Regarding postulate 2, this equals to restricting the number of real-worlds to one and only one.

Definition 1.9

p-morphism

> Let $\mathcal{E}$ be an environment, $\mathcal{P}_1 =< \mathcal{I}_1, \rho_1 >$ and $\mathcal{P}_2 =< \mathcal{I}_2, \rho_2 >$ perceptions over $\mathcal{E}$. The mapping $h : \mathcal{P}_1 \to \mathcal{P}_2$ is a *perception morphism* (p-morphism) :iff h is a mapping between the connotations $\mathcal{I}_1$ and $\mathcal{I}_2$ which is *no-blur*, i.e., the definite truth values (**t**,**f**) are preserved by the p-morphism.

As represented by Arzi-Gonczarowski, p-morphisms allow to describe translations between perceptions, their structural properties as well as effects on the meta-level like learning and the incorporation of individual perceptions in perceptions of a social group or culture. Further, they are the categorical morphisms of the category based on the collection of all perceptions with the same environment $\mathcal{E}$ (this set is denoted by $\mathcal{PRC}_\mathcal{E}$). Lemma 1 of [Arzi-Gonczarowski & Lehmann 1998a] proves that $\mathcal{PRC}_\mathcal{E}$ together with p-morphisms is really a category.

category $\mathcal{PRC}_\mathcal{E}$

1.8.2 Why Category Theory ?

Until now, this approach only covers an excerpt of the previous cognitive modelling, but it demonstrates the benefits of utilizing category theory[25]: based on the above categories, all the constructions of category theory can be applied to perceptions, e.g., coproducts to join perceptions with common extension and pushouts to generate common-sense connotations; thus allowing to underpin the actions taken on perceptions with a *simple*[26] formal foundation which is given free of charge by the possibilities of category theory [Goguen 1991].

[25] John Macnamara et al. argue in favour for utilizing category theory as foundation for models of cognitive science [Macnamara 1994b];

[26] simple when compared to an approach which needs to introduce these complex constructions from scratch without a proof of the correct outcome – both is implicitly given with categorical constructions;

Example 3 : Applying Categorical Constructions to Concepts

> Fig. 1.9 which is inspired by [Healy 2000, fig. 2] shows a possible co-limit construction. The basic object is a «cork» which has (p-)morphisms to both «cork in a bottle» and «cork with corkscrew». (Leaving aside the construction of these combined objects via co-products or some similar operation.)

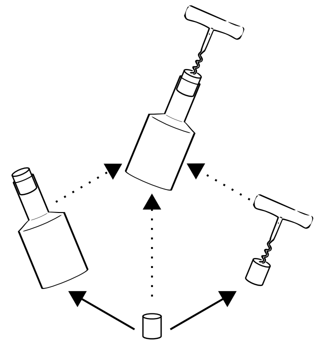


Fig. 1.9:
The Co-Limit Generation of a Composed Object
(arrows show morphisms between objects whereas dotted arrows propose the existence of morphisms)

> Now the co-limit generates a fourth object which combines all three as «cork in a bottle with corkscrew» such that the resulting diagram is commutative. Hence, one can generate relatively complex objects – think of the structural constraints regarding the position of the cork – in a single step of construction.

Consequently, the previous cognitive approach could be reformalized with the help of category theory; this would allow, for example, to mathematically prove the correctness of the proposed search algorithm.

Further, the category $\mathcal{PRC}$ and the idea behind p-morphisms has a notion similar to the foundation of formal languages as will be discussed in sect. 2.4; this would allow to contrast categorical semantics of perceptions with its counterpart for logic[27]; and maybe allow to underpin the semantic framework of the next chapter.

Additionally, category theory's inherent notion of (commutative) diagrams→ and the corresponding proof-method of diagram-chasing are important examples of formal diagrammatic notions with a formal semantics (cf. ch. 7) – which will later be called abstract logical diagrams.

These ideas will be readopted in chapter 8, which will extend the categorical keynote beyond $\mathcal{PRC}$ to include symbolic representation as well as new forms of semantic relations that enter the discussion with pictorial representations.

[27] a classical example is the usage of category theory for denotational semantics of linear temporal logic which is 'equivalent' to FOPC [Fiadeiro 2005, sect. 3.5];

2 Semiotics, Semantics, and Semantology

2.1 A first Approach to Signs

In order to approach pictures as signs, some basic semiotic definitions are inevitable. These will be based upon Charles Sanders Peirce's approach because of its focus on the dynamic creation of understanding and the production of meaning as opposed to an essentially static relationship between – in de Saussure's terms – the signifier and the signified.

> " *A sign, or representamen, is something which stands to somebody for something in some respect or capacity. It addresses somebody, that is, creates in the mind of that person an equivalent sign, or perhaps a more developed sign. That sign which it creates I call the interpretant of the first sign. The sign stands for something, its object. It stands for that object, not in all respects, but in reference to a sort of idea, which I have sometimes called the ground of the representamen. 'Idea' is here to be understood in a sort of Platonic sense, very familiar in everyday talk; I mean in that sense in which we say that one man catches another man's idea, in which we say that when a man recalls what he was thinking of at some previous time, he recalls the same idea, and in which when a man continues to think anything, say for a tenth of a second, in so far as the thought continues to agree with itself during that time, that is to have a like content, it is the same idea, and is not at each instant of the interval a new idea.* "
>
> [CP, 2.228]

sign (Peirce)

interpretant

representamen
idea

This passage of Peirce's work is one of his most famous and important definitions of «sign» of which [Marty & Lang 1997] list 76 different in his œuvre. It proposes a genuine triadic relation among a representamen (sign), an object, and an interpretant. Further, being a sign depends on a person that is able to grasp the intended idea, i.e., to interpret it as an interpretant; therefore, *being used* as sign is the only constraint on any object to be sign. Hence, the question about the definition of signs becomes *When is an object a sign?* which automatically leads to the area of language usage or communication, respectively. This first, naïve interpretation of Peirce's definition above will be a satisfactory base to enter the domain of signs and symbols. The next section will present a more detailed analysis of Peirce's above approach.

When is an object a sign?

2.1.1 A more detailed Look at the previous Quotation and an Uml-ish Reformulation of the Peircian Meaning Triangle ∗∗

Approaching the above definition requires a proper knowledge of Peirce's tripartition principle which will be briefly circumscribed later when introducing the three sign classes symbol, icon, and index (p59). There are several arguable possible inaccuracies regarding the previous quote, particularly the introduction of both the underlying idea and the interpretant.

Regarding the introduction of the idea which influences the relation between the object and the representamen, one would tend to describe this idea as concept which underlies the sign's denotation or, regarding the later semantic framework, as sense. Classically, the semiotic triangle would include this concept besides the representamen and the object. But Peirce's metaphysical trichotonomy forces another relation to appear: both the representamen and the relation between the representamen and an object, which will be called «denotation» in the following, are related to a third entity, named interpretant, that determines this relation and which Peirce calls "thirdness" (see p59 or [Pape 2004]). This interpretant represents the implications of the denotation, e.g., "*an effect upon a person*" [Hardwick & Cook 1977, p80], as well as its foundation; the latter includes a connection between the interpretant and the grounding idea. But this interpretant also has the character of a sign, thus can be used to refer to the original sign, i.e., representamen, and its denotation relation; hence, the interpretant allows to apply strictly symbolical reference as in the case of the word-pattern "duck" with the concept «duck» that depends on the connection of a real duck object with its primary duck sign.[1] The introduction of an interpretant as third entity beyond the basic idea or sense might seem odd at a first glance, but this procedure extends simple denotational sense to an embedding in the larger context of communication, to the different ways of denotational dynamics, and to the formalization of a sign's interaction with the actions of an author or recipient.

semiotic triangle and Peirce's trichotomy

[1] a more detailed view onto interpretants can be found in [Peirce 1983, ch. IV.3] and his letters to Lady Welby [Hardwick & Cook 1977];

The following diagram tries to recapture Peirce's definition with the help of Uml class diagrams. Anticipating the later discussion of modelling relations, relator, and roles with Uml in sect. 13.3.3, the diagram in fig. 2.1 refines the simple notion of a ternary sign relation.

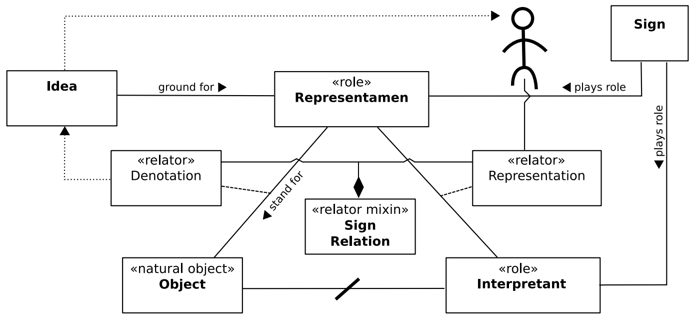


Fig. 2.1:
The Meaning Triangle

[2] for example, the object does not participate directly in the relation but plays a role; thus, this diagram is lacking the formal foundation of sect. 13.3.3;

The important step is the modelling of representamen and interpretant as roles which allows one (natural) sign object to take several of these roles simultaneously towards other objects. As can be seen, Peirce's original equivalence or is-a relation (sign equals representamen & interpretant is a sign) is transferred to the relation between a role and its player, as Peirce allows "*anything to take the place of a sign*"[Peirce 2000, ch. "Einleitung"]; hence, leaving behind Peirce's original definition, 'Sign' is not taken as a rolename but as an entity-type which explicates certain properties, i.e., a sign is some entity that can play the role of representamen.[3]

[3] this mirrors the question "When is an object a sign?", which influences the choice of appropriate player universals (p133);

The dependence (⋯⋯➤) of the denotation (UML-)relation on an idea depicts the influence of the ground of the representamen onto the relation towards the object which was not made clear by the previous definition, as well as the correspondence of ideas and interpretants. Peirce's original intention of secondness is recorded with the help of the denotation relation and not by the object itself. The given model can only be seen as a rough, first draft, as – in order to grasp Peirce's understanding of the sign relation – it must include the three participant's roles in his ontology as "firstness", "secondness", and "thirdness"[4] which loose their dynamic aspects when being depicted in a static class diagram as above. Another design decision is to model the central sign relation as a mixin→ of the other two relators whereas the third relator between Object and Interpretant can be derived ("/") from these two.

[4] see notes at the end of this part, and the brief introduction in in the discussion of icon, index, and symbol at sect. 8;

Besides the incompleteness of the above diagram, a profound treatment of Peirce's sign relation would require to include all his considerations which in turn are spread over his entire œuvre due to his distinct style of publishing results; at least, the above diagram serves as an example of rendering a conceptual model that is available in a linear, semi-formalized form into a formalized diagrammatic representation. Chapter 9 will resume some of these ideas in the larger context of modelling pictorial representation with GFO.

2.2 Communication

In the eyes of Dirk Baeker, communication is *the* theoretical construct of the 20th century [Baeker 2005]. Due to the vast field of diverse phenomena to which this concept has been applied, the question about the role of signs and meaning commences with a first working definition which will be refined consecutively.

Definition 2.1 — communication

> "*We might say that communication consists of transmitting information from one person to another.*"
>
> [wp:Communication_theory <200608121352>]

Ergo, an act of communication involves two agents (a sender and a receiver)[5] and at least one channel which transmits information. This is a bottom-up view onto communication, based on Shannon's famous attempt to ground information→ on statistics and to model it with the help a simple data-flow [Shannon 1948]. The downside of this mathematical foundation becomes manifest in its inability to describe communication as social interaction [Luhmann 1987, chapter 4], or to regard its primarily deontic purpose [Searle 2006]. Subsequently, a straightforward model will be introduced which allows to derive the communicative properties of pictures.

[5] other forms of communication, e.g., broadcasting or mass communication, can be reduced to this simple case;

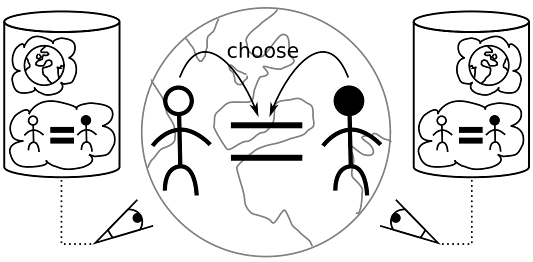


Fig. 2.2: Communication's Initial Situation

Communication takes place in a situation that involves at least two participating (human) agents (figure 2.2: ,) and some information that they are going to share over a channel (=). Both are embedded in an environment () and perceive themselves in their act of communication as well as the surrounding world (due to postulate 4 their world view is nevertheless different). All these percepts are part of each agent's contextual knowledge base; the agent is aware of his situation and hence able to directly influence the communication. They both choose, deliberately or not, a medium or an information channel, respectively. But this is only the starting point of the interaction which takes place in an act of communication.

The purpose of communication is not only the interchange of information but of meaning, i.e., meaningful pieces of knowledge[6], e.g., facts or commands. The crux

exchange of meaning

[6] to avoid at least one of these holy grails of science, «knowledge» will not be defined further and be taken in its common sense as "meaningful pieces of information";

of communication lies in the en- and decoding of these facts into as little information as possible, assuming economic constraints in the background. Tor Nørretranders proposed a simplistic model describing this process which is depicted in fig. 2.3, combining several ideas and figures from [Nørretranders 1997, ch. 5–8].

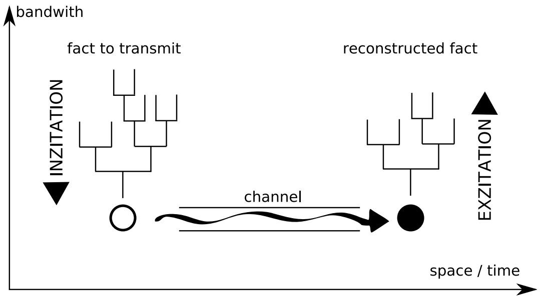


Fig. 2.3: Nørretranders's Tree of Knowledge

The entire act of communication gets embedded into a (commutative) diagram→ with an underlying spatio-temporal dimension. Without loss of generality, the transportation of a single fact (a small piece of declarative knowledge) should be explained. As the transmission of a fact, e.g., one can think of a Wittgensteinian "state of affairs" [Wittgenstein 1961], is restricted by the throughput capabilities of the channel (bandwidth, finite amount of time,...), the sender has to boil the fact down to some piece of information which is then transferred. This is done by *inzitation*, i.e., the computation (⅃) of information by eliminating background knowledge and knowledge common with the receiver. Therefore, inzitation transforms a meaningful fact into (statistical) information which can be divided into (a) the information to send and (b) *exformation* which is not transferred via the channel and must be recalculated by the receiver with the operation of *exzitation*. The operations inzitation, sending via a channel, and excitation form the communication of this single fact from one agent to the other.

inzitation

exformation

exzitation

Definition 2.2

meaning (communication)

> Meaning is can be described as the exformation in the communicative act, i.e., the not transferred information. [Nørretranders 1997]

With Shannon's definition of information→, the amount of exformation becomes measurable and therefore allows to empirically investigate the channel/media's information content and the ratio of sent information to exformation. The measure of information will serve as an important touchstone when comparing pictures and natural language as media.

After this excursion to communication theory, the question about the role of a sign in this social interaction will be focussed. Incidentally, the above description of a communication's content and transport by statistical means leads to an area already discussed when emphasizing the role of the underlying media and their influence on the entire process of transfer. Marshall McLuhan's description of communication as the difference of a pattern and its background [Molinaro et al. 1987] connects Gestalt theory and the statistical measures that underlie the building of equivalence classes of marks which compose a sign[7]. This resembles the idea of fuzzification of def. 1.5 and fig. 1.7, as the process of classifying marks equals the given pattern matching idea.

from marks to signs via patterns

[7] this step will be discussed later with notation systems at p30;

Assuming signs as major constituents of the information flow over the channel, another distinction becomes obvious. On the one hand, signs are the outcome of the process of inzitation, or in other words: a sign's meaning is *created* by the sender (author). On the other hand, the receiver (recipient) *derives* meaning from the sign. Umberto Eco therefore pleads for two different theories of semiotics in [Eco 1976]: a theory of the creation of signs (he denotes it by the term "communication") and a theory of codes ("signification").

2 ways of deriving meaning from signs

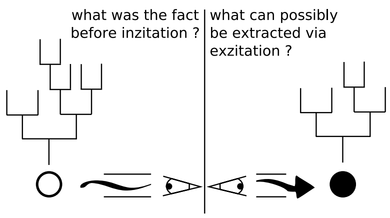


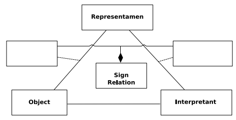
Fig. 2.4:
Two views onto Communication

This reveals an important issue when dealing with meaning: as there is no one-to-one transfer of knowledge because of the role of exformation which is derived differently by the author and the recipient, the meaning of the same sign in the same act of communication can be grounded in two different ways. Consequently most approaches that are to be introduced in later chapters, e.g., the two ways of reading the layered model of pictorial representations (sec. 5.3), can be divided into two major methodological categories by these two different points of view.

To summarize, the role of the interpretant in semiotics depends on communication and therefore the underlying media as well as a pragmatic context in which the transfer of information takes place. These are the reasons for the scientific dominance of the "simple" notion of sense (or idea, as Peirce calls it) over a formal approach towards the general interpretant. Nevertheless, the interpretant influences the relation between the sign and the denoted object which will be focussed in the next section.

2.3 Semantics

Recapturing Peirce's definition of signs (p23), the semiotic interaction is a triadic relation, with representamen, interpretant, and the 'idea of some object' as its corresponding relational roles→ (cf. fig.2.1). Projecting the ternary sign-relation to its three subrelations leads to the well-known semiotic triangle. Restricting the focus to the relation between representamen and the object, the sign's (relational) role is that of a *label* towards the object. Therefore, this sub-relation can be described as *labelling* . The application of this general term to describe the sign-object-relationship allows to describe and compare its different theoretical conceptualizations, e.g., as denotation, reference, and exemplification. To avoid the terminological confusion mentioned in [Materna 2004], the opposite role to 'label' will be denoted by the general term "*meaning*". Fig. 2.5 depicts the embedding of this relation in the framework of the previous sections.



(fig. 2.1)

labelling relation

meaning

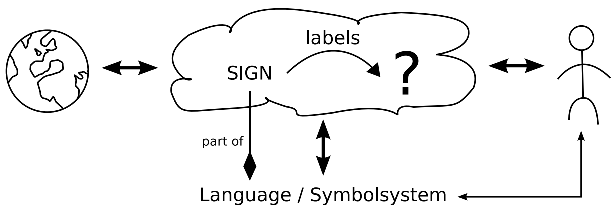


Fig. 2.5:
Sign's Dependencies

In accordance with section 2.2, the labelling takes place between an agent and its environment. The sign itself is not an independent entity but part of a symbol system [Goodman 1968] which is a cultural construct of an agent. The choice of a certain language framework restricts the kind of labelling as will be seen in the following discussion. As pictures are signs, the following examples and definitions are applicable not only to classical sentential languages which dominate most of the following examples, but to pictures and diagrammatic representation systems as well.

symbol system

Emphasizing the role of the symbol system, i.e., a culture's language, surmounts Peirce's triadic relation and the determination of a sign via a single interpretant which becomes a special case of this extended theory. Labelling *must* be considered in the context of a certain symbol system and therefore the cultural embedding of the language users. Nevertheless, Peirce's triad is a simple and elegant way to describe semiotic relations by concealing the social and cultural complexity.

The labelling relation itself is highly controversial. Semiotic theories can be compared by their different approaches to this basic and simple labelling link between a sign and its meaning. But before semantic theories will be introduced and contrasted, some further definitions are to be stated which allow for the introduction of the labelling relation as prior to sign and meaning.

Definition 2.3 semantics

Semantics is the study of the way of defining the labelling relation of a sign.

Definition 2.4 meaning of a sign

The *meaning* of a sign is its image[8] under the labelling relation. This can be an ‘idea’ or an object in the sense of Peirce (cf. p23) as well as a real-world object.

[8] in the mathematical sense: to what the relation maps to;

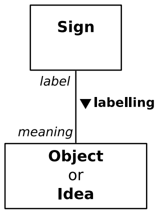


Fig. 2.6:
The Labelling Relation

Definition 2.5 sign, (Goodmanian) symbol

A *sign* is an entity that is mapped via a labelling to its meaning. Signs are no isolated entities, but are embedded into a symbol system.

[Goodman 1968]

Definition 2.6 symbol system

A *symbol system* is composed of (a) a domain of signs and rules to compose complex signs from basic ones (*syntax*), an additional semantics, i.e., (b) a domain of possible targets of the labelling relation, and (c) the relation between the signs and the corresponding object/idea especially considering the syntactic composition and the corresponding semantic consequences. [Goodman 1968, ch. 4]

syntax

composition

[Goodman 1968] introduces the stricter concept of a notation system which exceeds the definition of symbol systems by additional demands. Notation systems, by being the most rigorous type of symbol systems, play an important role in the context of automatic language processing, data storage, and formal languages.

Definition 2.7 notation system

A *notation system*, or simply *notation*, is a symbol system whose labelling relation is one-to-one.

Further, there are requirements regarding the class-comprehension of syntactic and semantic elements: the members of these classes have to be *disjoint*, i.e., there is no common element between different classes, as well as *finitely differentiable*, i.e., they are neither dense[9] nor is there any infinitely small difference between two members which results in the existence of a decision procedure that finds the corresponding class for a given syntactic or semantic element.

[9] mathem.: a (partial-) order $(A, <)$ is dense iff for any choice of two different entities $x, y \in A$ that are $x < y$ there is an additional element of the set $z \in A$ in between $x < y < y$; regarding conceptual spaces, maybe topological density would be a more appropriate notion;

The situation is depicted in fig. 2.7 which emphasizes the five necessary conditions for notations; four are given in a negative form, i.e., by the absence of some property. The idea of forming syntactic or semantic equivalence classes is an abstract notion of the cognitive mappings of different marks to a character and, dually, the class-comprehension of different semantic entities which Goodman calls *compliances*; in the language of chapter 1: the matching of different percepts to the same concept and the possibility of concepts to include different combinations of properties.

compliances

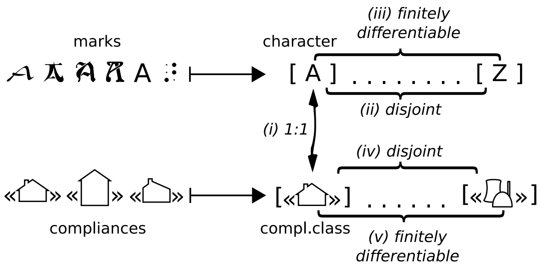


Fig. 2.7: Requirements to Notation Systems: Basic Act of Classification (Examples: Letter 'A' and Concept «House») and the five Requirements due to Def. 9

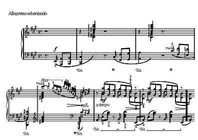
Example 4 : Notation Systems

This example is based on [Goodman 1968, ch. 5] and will explicate a notation system's five necessary conditions with the help of three classical examples: a musical score[10], natural language that will be introduced in the next subsection, and pictures.

[10] cf. the prototype of sheet music in ch. 6.1;

Musical Score

A musical score is a complex character with disjoint marks (notes, repetition signs, etc.). The compliance of a musical score is a singular performance. An opus collects different performances with the same underlying musical score. Leaving aside possible criticism against this semantic foundation [Winget 2005], the performances are disjoint, as one performance belongs to only one opus, and is one-to-one due to the fact that one performance's authenticity depends solely on the underlying score. Assuming a basic fuzziness – a certain sonata's performance remains the same opus even if a player misses a note in a certain performance – there is a decision procedure to classify performances as well as the supporting musical score. Hence, by fulfilling (i) to (v),



(from p 70)

a musical score with the compliance classes of the corresponding opera form a notation system.

Natural Language

Assuming a standard symbol system to express natural language, e.g., the standard English typeset with Latin letters, semantics is the crux of natural language due to the inherent semantic fuzziness, as well as its homosemies (hence not *(i)*), polysemies (thus, not *(iv)*), and a semantic foundation that is based on individual experience which renders semantic differentiability in general almost impossible (not *(v)*). Controlled natural languages→ which try to restrict natural languages to a simple, decidable[11] subset propose a way out of this dilemma.

< . . . . >
< . . . . >
< . . . . >
< . . . . >
(from p 70)

[11] decidable in the computer science sense: there exists a decision procedure; especially regarding the labelling relation's outcome;

Pictures

Finally, in the eyes of Goodman, the main property of pictures is their *density* which results in the importance of the smallest syntactic and semantic difference between two pictures. Further, Goodman emphasizes the problems of categorizing pictures as either marks or characters, as pictures are *autographic*, i.e., a copy of a picture is a *new* mark of another picture-character, as opposed to letter's *allographic* behaviour, e.g., a copy of the letter 'a' results in another mark of the *same* character. Consequently, pictures in general are far from being a notation system. The idea to restrict pictures and pictorial presentations to notational systems is part of sect. 7's introduction of diagrammatic representation systems.

dense

autographic

allographic

		syntax		semantics	
	1:1	disjoint	diff.able	disjoint	diff.able
(musical) score	yes	yes	yes	yes	yes
natural language	—	yes	yes	no	???
pictures	—	no	no	no	no

Fig. 2.8: Summary of Example 4
(conditions that are not applicable are marked '—', the answer '???' refers to a question that has no definite answer)

Framework to Classify Different Approaches to Semantics

Sign and meaning are only the relational roles→ of the labelling relation. Further, the socio-cultural embedding of symbol systems links the discussion of semiotics with languages and therefore ritualized labelling relations. Following [Hausser 2001, ch. "Semantics"], there are three major paradigms of symbolic systems corresponding to different ways of defining the labelling relation.

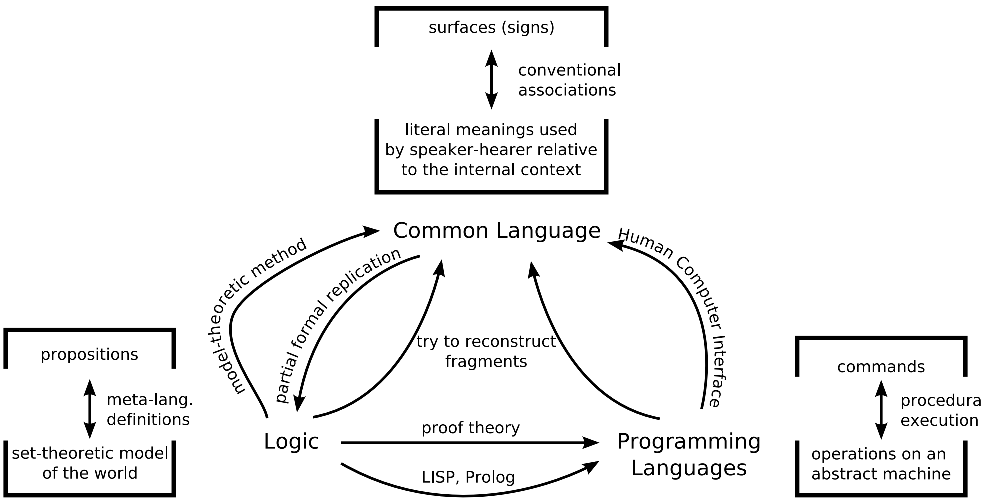


Fig. 2.9:
Three Paradigms of Symbolic Systems due to [Hausser 2001]

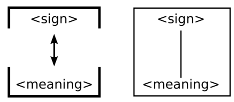


Fig. 2.9 combines several ideas from [Hausser 2001, p374ff]. The labelling relation is depicted by the top-down square brackets (in later examples: a box) and an arrow (line) connecting the relata.

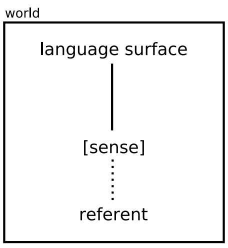
The first paradigm is (a) "Common language" or natural language, i.e., the language used in everyday's communication, which is heavily based on socio-cultural conventions and the subjective context of the speaker-hearer (see section 2.2 and the two ways to define meaning); another aspect is this language's dynamic evolution of semantics, e.g., etymology and the usage of metaphors. This is partially solved by (b) (formal) logic which originated in the 19th century attempt to partially formalize natural language and which uses a fixed a priori set of semantic relations which map into set-theoretical models; in its classic (Fregean) approach, which will be discussed later, the labelling relation can be expressed completely in a meta-language. Finally, (c) programming languages can be seen as today's implementation of case (b)'s ideas; their importance relies heavily on the usage of computers and the need to interface computers from the level of common language. Another way to grasp the above distinction is by classifying the possible meanings of natural language into static, propositional knowledge (as handled by logic) and dynamic commands. Therefore, (b) and (c) try to formalize as much aspects of common language as possible.

applying the paradigms to pictorial representation

Before going into more detail, the upper triad can also be seen in the context of pictorial representations: there is (a) a common language of sketches, pictures, etc. which is used in everyday's communication; classical examples of logical diagrams, e.g., Venn diagrams or Sowa's conceptual graphs (see sect. 11), allow to express propositions in a graphical way with an underlying set-theoretical model analogously to (b); visual programming languages [wp:Visual programming] form a simple example for case (c)'s mapping of commands to operation ("operational semantics").

Hauser's 4 basic ontologies

To avoid the dynamics of common language usage, now, the following investigations will focus on the formal languages's approaches (b) and (c) and categorize the labelling relation in more detail. Hausser suggests four modes or "four basic ontologies of semantic interpretation" which will be presented in the following [Hausser 2000]. These modes correspond to the presence or absence of sense as an additional meaning-bearing layer and the question whether the model-structure is part of the communicating agent (constructivistic view) or whether the agent is part of the model structure which will be depicted by different inclusions of the labelling relation in the boxes. Consequently, the four ontologies will be named by the scheme "[± sense, ± constructivism]" and will be depicted in the graphical notion introduced above. The following discussion and the diagrams are mainly based on [Hausser 2000] and [Hausser 2001].



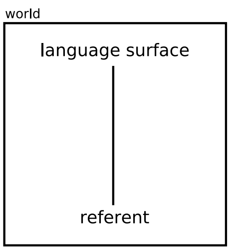
[+sense,-constr.]

[+sense, -constr] : Fregean Approach

The first mode corresponds to the classical Fregean semantic tradition originating in [Frege 1879], [Frege 1884], and [Frege 1892]. Meaning is split into sense ("Sinn") and reference ("Bedeutung"). In absence of any constructivistic tendencies, the reference relation maps to objects of the real world. *Sense* is best introduced via the classical example of an unicorn – there is no possibility to refer to a real world object, but a sentence like "Nobody has ever seen a unicorn" is nevertheless meaningful. In this tradition, senses are newly introduced entities belonging to a realm of reality Frege called the Third Reich. Further, senses influence the mapping to reference objects. As already proposed in fig. 2.9, the labelling relation is a priori stated in some kind of meta-language.

sense

unicorn example

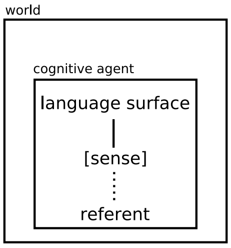


[-sense,-constr.]

[-sense, -constr]

One possibility to hide this sense layer which is nevertheless indispensable due to the above unicorn example is offered by [Carnap 1988]. An expression directly labels an index function (intention) that allows to find reference objects (extensions) in worlds of an underlying multiple world model-structure and in one possible world which is indexed by the intension belonging to the term 'unicorn' there exists an object which is an unicorn.

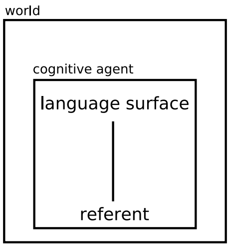
multiple possible worlds



[+sense, +constr.]

[+sense, +constr]

In analogy to chapter 1 and its postulate of embodied intelligence and constructivism , the two upper modi leave out the role of the cognitive agent who builds the model of the world. However, the introduction of sense is still important because of common languages's copious usage of abstract objects and metaphors. This approach would allow to describe common language usage, though the complexity of the required description is far beyond the other approaches.



[-sense, +constr.]

[-sense, +constr]

In the case of programming languages and robotics, the internal model plays the central role since the connection to the outer world can be described by a simple sensor-actor-characteristic or an input-output-behaviour of an (abstract) machine. These both allow to handle [+constr.] efficiently; contrariwise, an additional level of sense would only increase the number of entities and should therefore be avoided (Newell & Simon due to [Hausser 2001, p399]). This restricts the application of this mode to simple domains like the famous blocks world [wp:blocks world].

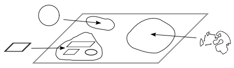
from "+" to "-"

Comparing the above four modes, the reduction of the 'plus' states to their counterpart seems relatively easy: [-construct] is a special case of the constructivistic view in which the entire external world is isomorphically represented internally, therefore a distinction between these two fields of experience becomes unnecessary; as seen with Carnap's idea and Newell and Simons argumentation, the renunciation of sense simplifies the model, i.e., there are fewer objects to handle in the semantic domain, whereas the hidden complexity of the underlying model increases.

The Role of Truth ∗

With [Hausser 2001], another difference can be envisaged: the possibility to handle vagueness and the role of truth. (Conceptual) vagueness is a main feature of common language. Referring to the idea of a concept space, the borders between concepts, i.e., the responding set of properties, are either unstable and change over time or are ambiguous (cf. fig. 1.4). Modelling this language feature is not possible in [-sense,-constr.] approaches because the labelling relation has to be fixated a priori in a meta-language in an "unambiguous" manner [Tarski 1983b]. Accordingly, a change of the semantics requires a change in the underlying model and therefore introduces a radically new view of the world. Serendipitously, the [+sense,+constr.] approach

(conceptual) vagueness



(fig. 1.4)

can adapt to this vagueness because the labelling is done as pattern matching[12] inside the agent on the basis of the constructed model (compare to post. 4). The labelling relation becomes purely dictated by pragmatics, i.e., the usage of the language in a certain context.

[12] cf. the cognitive algorithm of sect. 1.5;

absolute truth

Fig. 2.10 compares these two extremes with respect to their handling of truth. The starting point is the relation of a sign, e.g., an assertional sentence, to a fact or state of affairs of the world. The [-sense,-constr.] approach is simply the Tarskian meta-language way to define the truth of the sentence. In [Tarski 1944] and [Tarski 1983a], Tarski starts with the example "*The 'Snow is white' is true iff the snow is white*" and introduces a recursive definition of truth based on the correspondence of elementary expressions, e.g., "a name 'X' (of an entity) is true iff there exists a corresponding entity", and a way to construct the truth of complex expressions analogously to their composition out of simpler expressions whose truth, i.e., correspondence to reality, is known due to the recursion of this method. Later chapters will refer to this paradigm as principle of compositionality (def. 2.9). Herewith, the strong tie of [-construct] approaches to truth becomes clear: truth can be defined directly by reference to facts of the world and therefore is an usable measure for the quality of sentences.

compositionality

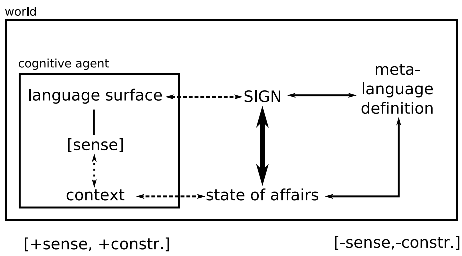


Fig. 2.10: Comparing [+ +] and [- -]'s Handling of Truth [Hausser 2001, p405]

contingent truth

However, a cognitive agent can access the state of affairs only through its internal representation as "context"[13]. Truth becomes a quality of the internal mapping of the perceived sign, i.e., the assertional sentence, to its meaning in a certain context. Therefore truth is no absolute quality of the world but becomes connected to a context and a pragmatic assignment by the agent. This view allows different definitions of truth, ranging from a cognitivized version of Tarski's correspondence definition to James's pragmatistic truth definition [James 1949]. Nevertheless, the role of truth in these semantic models becomes negligible compared to [-constr.] modes.

[13] the discussion of context forms a concurrent thread of discussion which is not elaborated exclusively, starting with the background knowledge in cognition to ontology-based semantics in later example 8;

To conclude, the reference to truth needs a reference to the underlying semantic model. Truth is *the* central notion in logic and most operational semantics of programming languages[14], but it should be emphasized that this notion of truth is rather technical compared with other possibilities to ground truth which are presented comparatively in [Skirbekk 1996] and [Kirkham 1992].

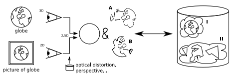
[14] as well as notion of quality, e.g., the application of the three truth-values **t**, **f**, **u** as evaluation of the perception predicate ρ in sect. 1.8;

2.4 Using the Semantics Framework

Consequently, the above framework allows to categorize different approaches of defining semantics. As an example, in the following, two attempts will be contrasted: the cognitive entry of 1 and the semantic model that underlies [Helbig 2001].

Example 5 : Conceptual Spaces Revisited

Obviously, the idea of conceptual spaces is [+sense,+cogn.] and allows to differentiate between semiotic objects (the picture of the globe in fig. 1.6) and objects which are the semantic base of concepts (concept «globe»). Both are perceived in the same way and the pattern matching algorithm tries to find a perception-equal conceptual representation. The crux lies in the handling of signs that are not "close" to perception, i.e., do not resemble the designated object by any means. Think of the English word "globe", which should be mapped to the concept «globe» which resembles the perception of a (real) globe . The word itself is mapped via cognition to the concept «5 letter word: g l o b e» assuming letters as basic patterns. As there is no way to define any equality-relation in the concept space between the properties belonging to these two concepts, the mapping of the word to the concept «globe»  must be part of the background knowledge that has to be learnt by the agent in his socialization, i.e., the learning of his culture's symbol system or language, respectively. To summarize, there exists a concept of the word "globe" which collects the perceptional impression of these letters and an artificial, socio-culturally dependent link stating a concept-equality of this word-concept to the concept which originated in the perception of globes.[15]



(fig. 1.6)

perception of words and their elation to concepts

[15] cf. the idea of complex symbols of example 6;

Section 1.4 tried to explicate a bottom-up approach to cognition and semantic interpretation. Now, a top-down approach will be introduced which resembles the classical main-stream model that underlies most of today's research in knowledge representation, logic, and linguistics. Helbig's model will demonstrate the different possibilities to interrelate signs and real objects [Helbig 2001, p19ff].

Example 6 : Classical Linguistic Approach

At a first glance at fig. 2.11, senses reside in an cognitive layer and not – like in the classical model-theoretic approach – in Frege's Third Reich. This is analogous to the above example. Nevertheless, these senses refer to objects of the real world and therefore this approach is prima facie [+sense,-constr.].

Leaving aside for a moment the new facet of formal knowledge representation, a word of the sentence is interpreted as *complex symbol*, i.e., a concept

complex symbol

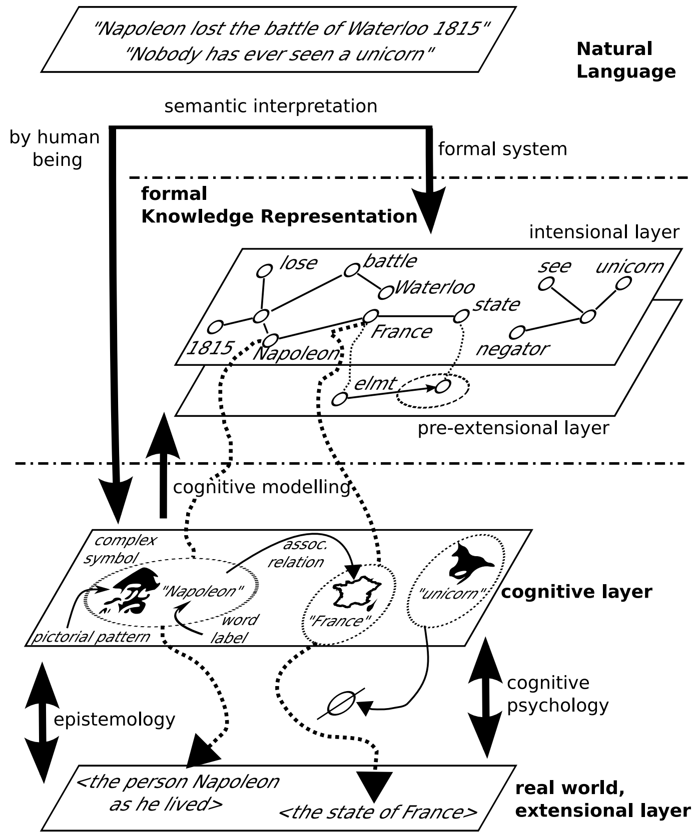


Fig. 2.11: Top-Down Approach: from the symbolic layer to the real world (from p19 in [Helbig 2001])

in the sense of def. 1.3 consisting of visual properties of a real object (the real Napoleon, a portrait of Napoleon) and a symbolic representation[16]. This resembles the role of the background knowledge in the previous example which stated the concept equality of the pictorial pattern and the word label. The labelling relation between words of a language and the corresponding senses is seen as part of the symbol system that has to be chosen before analyzing the sentence. Referring to the previous categorization of semantic approaches, this reminds of the logical approach to describe common language semantics.

[16] a sequence of letters or a pictogram (cf. p74) of his famous hat;

Formal Knowledge Representation

The interesting part of fig. 2.11 is the explicit reference to a formal knowledge representation language and its embedding in the context of natural language understanding. Thus, this example should be interrupted briefly for a closer look on formal languages.

Definition 2.8

formal language

A *formal language* or a formal language system is an artificially created semiotic system or notation system, respectively. Its basis is a fixed finite signature[17] σ that grounds syntactic rules *syn* to generate complex expressions from basic ones and an additional way of defining *formal semantics sem*, i.e., the model grasped by a formalized structure, e.g., a mathematical, relational structure, regarding the principle of compositionality (as will introduced later in def. 2.9). Hence, a formal language system is a triple (σ, syn, sem).

[17] also called "alphabet" or "vocabulary";

formal semantics

A formal language can be extended by two additional entailment relations: $\vDash$ symbolizes semantic entailment and is based on model-theoretic considerations; $\vdash$ describes purely syntactic entailment via rules of deduction (see fig. 2.12). Therefore a formal language system becomes a quintuple $(\sigma, syn, sem, \vDash, \vdash)$.

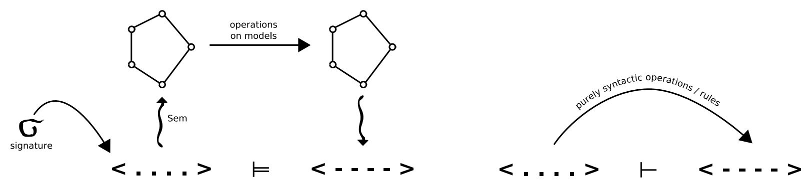


Fig. 2.12: Formal Language's Entailments

Formal systems already played an important role without being introduced from a formal language's point of view. The most prominent formal languages are logics, above all the first-order predicate calculus (Fol/ Fopc)[18] which is introduced in appendix B, and common logic; both are important tools to be used later in section 12.1 for the semantic foundation of conceptual graphs. Fol goes back to Frege's approach to state a new foundation of mathematics with a formalized notion of logic in [Frege 1884], introducing mathematical rigour into a field which was previously mainly dominated by philosophy. Formal language's semantics is innately [-constr.] and by the restriction to mathematical, abstract models (either the classical relational structures or their generalization in category theory) in most cases [-sense] with a strict separation of the labelling relation's domain of signs and the possibly meaning-bearing objects. A distinctive feature is the compositionality principle which describes the direct mirroring of semantics in syntactic composing rules as explicated in the following definition which is the essence of [Hausser 2001, ch. "Frege's Principle…"].

[18] one has to differentiate first order languages/logics and the first order predicate calculus (Fopc); "*In short, the notion of 'first-order logic' is independent of the subject matter or the notation. Predicate calculus is a mathematical notation for logic, which may be first or higher order.*" [Sowa 2007]; nevertheless, the more common abbreviation Fol will be used in the following where Fopc would be more appropriate;

Definition 2.9

compositionality

Following [Hausser 2001, p420], a semantic interpretation has the property of (surface-) compositionality iff

(i) the syntax is restricted to the composition of concrete expressions

(ii) semantics is homomorphous→ to syntax, i.e., the meaning of a composed expression is built on top of the meaning of the underlying expressions and their syntactic formation

(iii) there is no semantic mapping to both zero-elements (remember the unicorn in the above example) and identity mappings (a complex entity must denote strictly more than its parts) regarding homomorphisms.

Fig. 2.13 depicts how meaning is assigned to a complex expression in the case of a compositional (formal) semantics. By (i), one can straightforwardly assign a syntactic composition to a complex expression (< . . . . >). This is repeated recursively until a basic expressions with a concrete semantic mapping terminates this procedure; requirement (iii) assures the existence of these basic building blocks. Now these basic units are semantically mapped to the model structure[19] and the mapping of the complex expression is derived by homomorphically composing (ii) the meaning of the building blocks.

[19] the labelling relation is 1:1 (←→) such that one can start to reversely generate a syntactic expression for any given semantic compound;

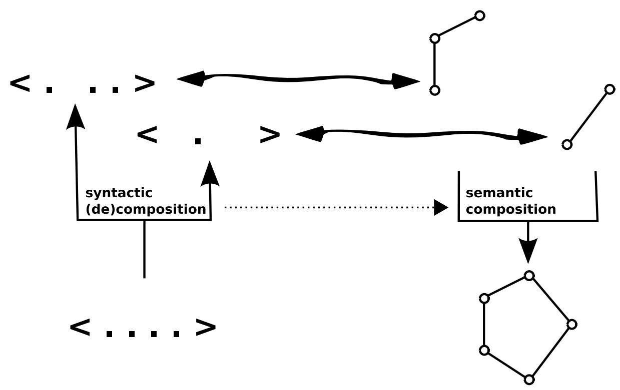


Fig. 2.13:
The Principle of Compositionality

Formal Languages are applied in knowledge representation (Kr) which can be introduced as formalized counterpart to everyday communication. A special kind of knowledge representation is conceptual modelling which will be discussed later. As the term Kr is often applied in a shallow, inflationary manner, the following enumeration only outlines certain of its aspects: a *knowledge representation* is (a) a surrogate, (b) a set of ontological commitments, (c) a fragmentary theory of intelligent reasoning, (d) a medium for efficient computing, and (e) a medium of human expression [Sowa 2000].

knowledge representation

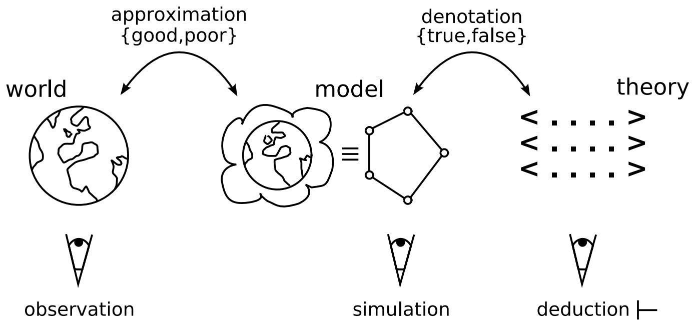


Fig. 2.14: World-Model-Theory (based on [Sowa 2001a, fig. 12])

knowledge engineering

As *knowledge engineering* is "*the application of logic and ontology to the task of building computational models of some domain for some purpose*" [Puppe et al. 2003, p601], the basic entities of Kʀ can be related as in fig. 2.14. The relation between an aspect of the (real) world and a model can be compared to the special case of building a mental model in section 1.5. This approximation of some part of reality can be evaluated in terms of the quality of the matching (here valued by {good, poor}). In Kʀ, this model is described by a formal theory, i.e., a list of sentences in a formal language[20]. In other words, a theory is labelled to (has the meaning of) a model. Abstract truth plays an important role due to the usage of a *formal* language, and is therefore a qualitative measure to the fitting of a theory and an underlying model. The important property of Kʀ models is the possibility to derive new insights by simulating the computational model, and to implicitly state the formal language's and the real world model's equivalence (≡) which allows to tie a theory to a statement about the world. This equivalence needs some further foundation, but first, the example demands further attention.

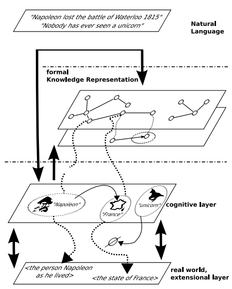
[20] cf. [Goguen & Burstall 1992];

truth in Kʀ

Example 6 : (continued)

With the introduction of formal languages and Kʀ , the semantic part of figure 2.11's mentioning of formal knowledge representation can be embedded into the previous discussion of common language semantics.

The sentences at the top of fig. 2.11 are stated in a *formal* language or a controlled natural language→ . Therefore, they can easily be translated into a [+sense] model, here called intensional layer. A classical formal language approach would stop now at an set-theoretic extensional model which would allow to describe set-theoretic relations like the subsumption of France under the concept state by the set-theoretic element relation. Helbig talks of an pre-extensional layer because the underlying model is not independent of a common language's cognitive sense layer and its (extensional) mapping to real world objects. By knowledge representation, this layer is bound to a certain

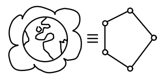


(fig. 2.11)

usage of this language, i.e., knowledge engineering's task of modelling a real world domain.

Considering the plain, formal KR approach, one often subsumes elements of the intensional layer under «concepts». This has its origin in Frege's introduction of "Sinn" and – recapitulating the unicorn example – the mapping into the Third Reich as associating the word 'unicorn' to the concept ("Begriff") «unicorn». Regarding the reduction of [+constr.] to [-constr.] approaches (cf. sec. 2.3), these KR concepts can in [-constr.] semantics regarded as reductions of the cognitive based concepts of sec. 1.3. To avoid this confusion, in this thesis, concepts are used solely for entities of the cognitive layer. A more detailed approach to this homonymy is given in appendix C at p166.

At this point, two important questions are left open: (a) how to ground the proposed equality of fig. 2.14, and (b) how to relate two different semantic approaches as seen in the example above. Serendipitously, these two questions are interrelated and can easily be approached by the missing investigation of conceptual modelling.



(fig. 2.14)

2.5 Conceptual Modelling

Conceptual modelling, as a formalized technique of modelling concepts, is the missing link between cognition's conceptual spaces which were introduced in sec. 1.3 and formal knowledge representation. First, conceptual modelling and the appropriate formal language framework will be introduced. Second, the semantic mapping of conceptual languages will be explained with the help of the already introduced semiotic framework.

Definition 2.10 — conceptualization

"*A conceptualization [is] the set of concepts used to articulate abstractions of state of affairs of a given domain. The abstraction of a given portion of reality articulated according to a domain conceptualization is termed here a [conceptual] model*" [Guizzardi 2005, p2]. — conceptual model

In order to use or communicate conceptualizations, a semiotic representation is necessary. As the compositionality underlying a concept space can be adequately represented by a formal language's compositional semantics, a formal knowledge representation language will serve as the basis for this communication (sect. 2.2). The set of previously shared background-concepts and their relation, which are required for the ex- and inzitation, form the basic units of semantics – an idea that will be

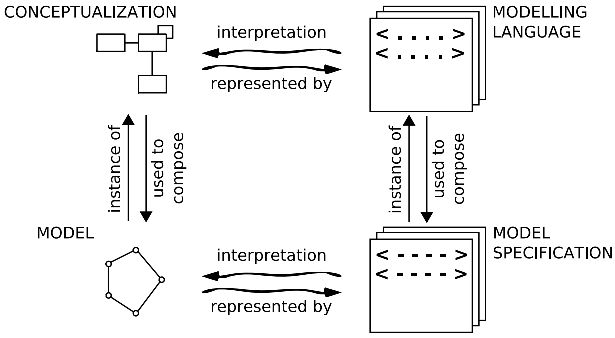


Fig. 2.15: Conceptual Modelling Languages (due to fig. 1-1 of [Guizzardi 2005])

used later to introduce formal ontology. Nevertheless, conceptual modelling is tied to cognition via the usage of concepts and depends on a finite set of background knowledge that was shared beforehand.

John Mylopoulos's definition of conceptual modelling in [Mylopoulos 1992] shows the mentioned subsumption under knowledge engineering and restricts the scope of application even further to real world (material) entities and the social stratum.

Definition 2.11

Conceptual Modelling

> Conceptual modelling is "*the activity of formally describing some aspects of the physical and social world around us for purposes of understanding and communication [...] Conceptual modelling supports structuring and inferential facilities that are psychologically grounded. After all, the descriptions that arise from conceptual modelling activities are intended to be used by humans, not machines [...] The adequacy of a conceptual modelling notation rests on its contribution to the construction of models of reality that promote a common understanding of that reality among their human users.*"
>
> [Mylopoulos 1992][21]

[21] cited due to [Guizzardi 2005, p3];

Avoiding the discussion of the ontological foundation of mathematical entities [Troelstra 1991][22], conceptual modelling can also be applied to mathematical domains whereas modelling with the help of the axiomatic deductive method is the most common way of introducing mathematical concepts.

[22] hence, the question whether mathematical entities are part of the physical world;

After this excursion to conceptual modelling, the question about the semantic foundation of formal KR systems can be restated and tied to a concrete modelling paradigm: What is the connection of conceptual models as part of a formal KR language system and perception's concepts, i.e., how can conceptual models be grounded regarding a [+constr.] semantics?

the ≡-question

Fig. 2.16 connects Guizzardi's ideas of fig 2.15 and the [+constr.] approach of section 1.5. First, the bottom-up approach of building a conceptual language will be fo-

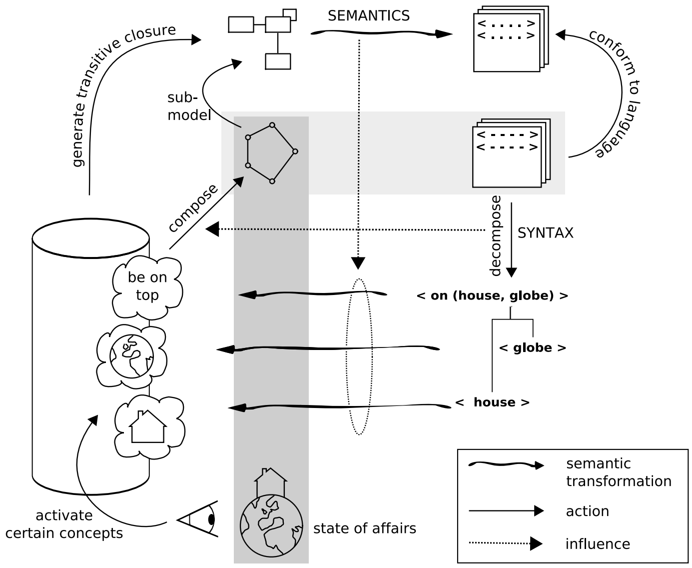
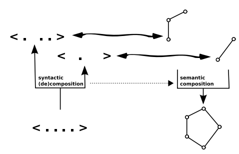


Fig. 2.16: Conceptual Modelling Semantics

cussed. The starting point is a certain aspect of the real world – a state of affairs[23]. As explained in section 1.4, these aspects are the origin for building an internal conceptual knowledge base, i.e., an agent's personal conceptual space. Following Guizzardi, one can build a kind of transitive conceptual closure over these concepts, e.g., combining each concept with each other according to every possible construction pattern. In terms of the cognitive search space approach, this is equivalent to building a complete model of the search space. The next step is to build a modelling language on top of that complete model whose semantic labelling relates expressions to parts of this model by a classical compositional approach: basic expressions are mapped to primitive concepts and their syntactic composition resembles the building patterns of the knowledge base. Which concepts are to be chosen as the primitive ones depends on the granularity of the conceptual modelling and can therefore not be fixed in an absolute manner.

[23] e.g., state of affair in the sense of Wittgenstein [Glock 1996];

Having chosen a modelling language, the embedding of a model specification, respectively a theory or list of expressions, can be semantically grounded. The syntax of the language describes the decomposition of expressions, and the semantic allows – relying on the principle of compositionality – to build a model structure which can then be regarded as sub-model of the complete model (cf. fig. 2.13). Consequently, the model's elements which are basic concepts (sub-concepts) are related to perception; therefore, they originate in real world objects and allow to ground parts of the model in the external world.



(fig. 2.13)

At this point, the next step would be to map concepts to their real world origin. But this procedure depends on the choice of the underlying semantic and cognitive theories. From the standpoint of [+const.] (compare postulate 4), only sub-concepts can be mapped to visual properties of real objects. Therefore the communication of expressions about real world objects depends both on a similar way of assigning concepts to percepts in each human and an arsenal of abstract concepts which have to be learnt as basic cultural skill. Hence, an one-to-one, extensional mapping of concepts to real world objects is only possible in [-constr.] paradigms like Helbig's from fig. 2.11 which reduces the cognitive layer to represent the entities of [+sense].

grounding conceptual modelling languages

Subsequently, three ways of semantically grounding conceptual modelling languages became obvious: (a) one stops at the abstract modelling level without connection to the conceptual grounding via cognition or (b) the model is based on concepts which are either the basic units of cognition ([+constr.]) or (c) only a step in-between towards real-world objects ([-constr]). An important interim solution is the combination of the semantics based on abstract mathematical models with additional conceptual grounding which is accomplished by special means of defining the modelling language and extending the framework with a formal ontology. This will be discussed in the next section's examples.

2.6 Formal Ontology in the Context of Semantics

Encompassing the different language formalizations of a conceptual model, the interrelation of languages has to be analyzed as well as the role of formal ontology. In this section, the basic Kr language is a formal language with [-constr.] and a set-theoretical extensional model structure, e.g., Fol. These languages are important in practical knowledge engineering because of the straightforwardness of their semantics and therewith the possibility to utilize computer-assisted deduction and model checking→ algorithms to support the engineering task.

To increase the quality of the interplay of ex- and inzitation in communication (p27), a fixed set of background knowledge, i.e., a predefined conceptual space, is necessary. When dealing with the archiving of knowledge and communication of non-human software agents, this agreement becomes inevitable.[24]

[24] especially the last point explains the on-going hype of buzzwords like "*semantic* web" or computer scientist's discussions about ontological foundation, an area which has been predominated by philosophy for the last centuries, as philosophy declares itself as the science that is mainly based on generating and scrutinizing conceptualizations [Deleuze & Guattari 1990];

The above framework (fig. 2.16) allows to introduce this agreement by stating a knowledge-base that is accessible to both communicating agents which contains the underlying conceptual space. The contained concepts are *inspired* by human cognition and therefore originate in the real world (basic axiomatization in chapter 1). These concepts build up what Guizzardi called conceptualization which is the base of modelling language's semantics. When talking about formal ontology one normally refers to a formal language which has a fixed set of concepts and their basic interrelations as semantic foundation; therefore, a formal ontology represents a dissection of the real world into concepts and their relations.

formal ontology

Again, this formal ontology also needs to be communicated in advance which leads to the problem of fixating its semantic foundation in another meta-language which, obviously, can neither be the language which depends on the formal ontology nor the formal ontology language itself. Therefore, the interrelation of different languages and their semantics gets into focus.

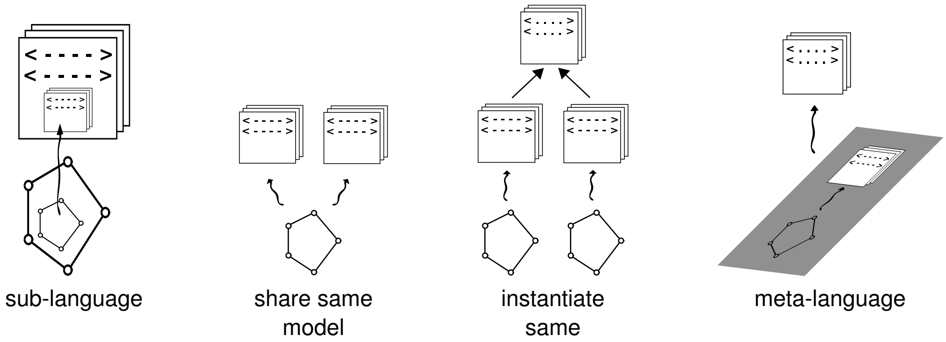


Fig. 2.17: Interrelation of two Languages regarding the underlying Models

There are several ways to relate different languages as depicted in fig. 2.17 which underlie later examinations. The most prominent interrelation is named by the relational role of the ‘upper’ language as *meta-language* approach; it was already used above when defining Tarskian truth by “*A sentence ‘<. . . >’ is true when <. . . > is true*”. This sentence talks about a sentence ‘<. . . >’ in an object-language (note the single quotation marks) which is part of the meta-language but exceeds it to talk about sentences in the object language and the definiendum truth which also is a meta-level concept. A comprehensive study of adding a meta level language to an object language in a language framework is done by Carnap in [Carnap 1988, p3f] from which only the *meta-meta-problem* will be explicated here. In analogy to the above problem of grounding a formal ontological language, one stops the meta-approach at a certain level circumventing an infinite hierarchy of meta-levels. Using the axiomatic deductive method, this quandary cannot be avoided in general, but disarmed to a level which is usable in practice.

meta-language

meta-meta-problem

Definition 2.12

axiomatic deductive method

> The *axiomatic deductive method* is based on a formal language system and its deduction rules. In order to avoid circular definitions when composing (mathematical) concepts on the basis of others, one fixes the ‘most basic’ concepts by axioms which are per se not based on other definitions.

Therefore, these axioms together with their deductive closure, i.e., all statements whose syntactic expressions can be deduced by the deduction rules ($\vdash$), form the fundament for the definition of ‘new’ concepts or the derivation of ‘new’ statements by mathematics creative technique of (abductive) proofs.

A famous example for this paradigm are Euclid's "Elements"[25]. Another example for the axiomatic deductive method is GFO's foundation [Herre et al. 2006, p7f] as introduced later in section 13.3.4 and appendix D. Recapitulating the recently introduced ideas, the following definition seems natural:

[25] Byrne's famous edition [Euclid 1847] uses graphical proofs only as discussed later (p99);

Definition 2.13

formal ontology

A *formal ontology* is given by an "*explicit specification of a conceptualization*"[Gruber 1993] in a formal language. In order to be communicated, a formal ontology needs a finite representation. Therefore it is stated as axiom system in a formal meta-language which has to be chosen as simple as practically possible without needing any further foundation.

Most formal ontologies aim for a taxonomic hierarchy of the basic concepts and therewith primarily express a subsumptive dissection of the real world, i.e., its cognitive internal construction depending on the pre-chosen semantic paradigm.

top-level language

Consequently, a hierarchy of languages can be grounded in such a top-level language that is not described on any further meta-language level but grounded in itself by the assumption of the existence of basic concepts qua axiom. Therewith other knowledge representation languages can rest upon one or a small number of top-level ontological languages which form the basis of communication and the storage of knowledge.

Two final and concluding examples explicate the possibilities to inspect semantic frameworks containing different languages and models as well as formal ontologies.

Example 7 : Relating Different Formal Ontologies

This example is taken from [Guizzardi 2005, p48, fig. 2-23] and shows the interconnection of different types of formal ontologies. The main idea is to translate from one formal ontology into another by using a more general domain ontology. 'More general' is here defined by model inclusion: the domain ontology was generated (in the above sense) from a material domain conceptualization which subsumes the models of the two formal ontologies. Furthermore the two sub-models relate to sub-languages of the domain ontology which can be seen as interpretations (in the language sense) i_1 / i_2 from FO_1 / FO_2 into the domain ontology which in this case serves as *conceptual interlingua*, e.g., a translation from FO_1 to FO_2 is now possible by

conceptual interlingua

$$FO_1 \xrightarrow{i_1} DO \xrightarrow{i_2^{-1}} FO_2.$$

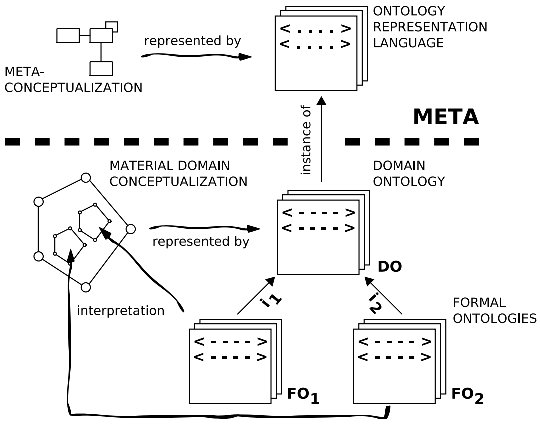


Fig. 2.18: Conceptual Modelling regarding Formal Ontology (due to fig. 2-23 in [Guizzardi 2005])

The above approach generates the domain ontology from a desired domain conceptualization which was built on top of models of both ontologies but the question is how to find an usable and small axiomatization of the domain ontology such that it can be communicated and reused as a general toolkit to translate between different formal ontologies stated in different formal ontology languages. Again using this approach, the translation and connection of different domain ontologies leads a step further in the abstraction hierarchy towards a *core ontology*. To avoid the classical meta-meta-problem, one normally tries to stop at this meta-level by the axiomatic deductive method. The question whether there can be only one single core ontology which subsumes all others is still unsolved (and maybe basically unsolvable). Today's discussions [Obrst et al. 2006] tend to assume a set of core ontologies which share some basic properties, ideas, and basic views. [26] A more abstract view onto the semantic foundation via formal ontologies is given in the next example.

core ontology

[26] see `http://ontolog.cim3.net/` for a virtual community that contains most of today's established ontology developers

Example 8 : Ontology-based Semantics

Guizzardi based his above example in Mihai Ciocoiu and Dana Nau's archetype of an *ontology-based semantics* [Ciocoiu & Nau 2000]. In addition to the previous example, this approach investigates the underlying formal languages of the formal ontologies. Moreover, it represents an example of Helbig's inclusion of a formal representation language to semantically ground sentences of a (declarative) language (example 15) and the interplay of mathematical model structures and cognitive background knowledge.[27]

ontology-based semantics

[27] and ultimately, the original sketch of the following figure can be used later as an example of an incon-cise, "bad" diagram as it bears certain design flaws and is surely *not* intuitively readable – which could not be solved entirely by the simplified version presented here;

Like Helbig's approach, this examples starts with a set of sentences S in a language K which then are rendered into a first order language. These translated sentences S' are basis of a logical theory in the formal language L because they implicitly allow to automatically deduce additional sentences by L's de-

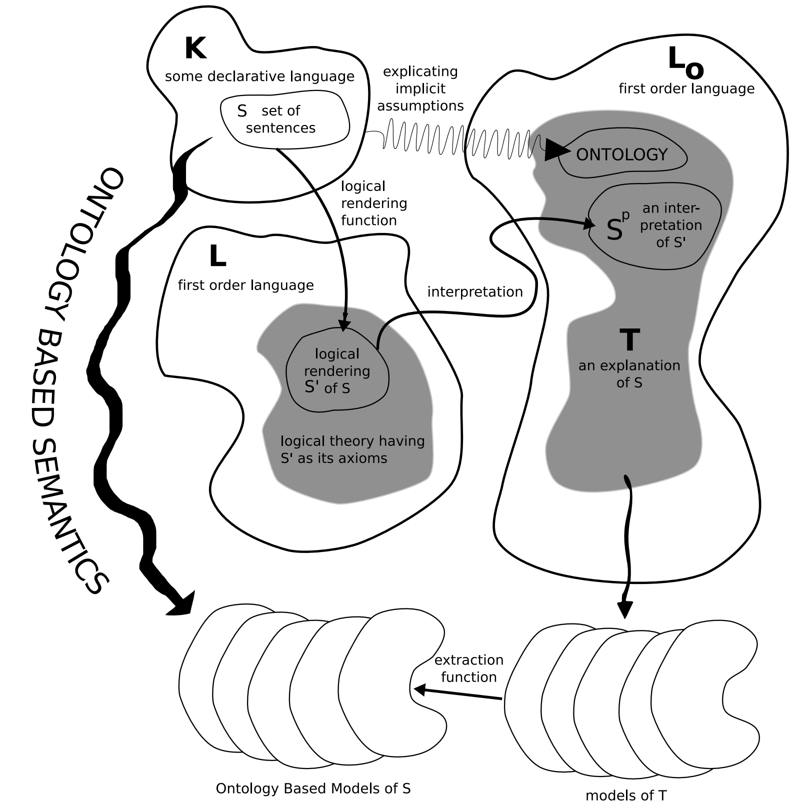


Fig. 2.19: Ontology-based Semantics (simplified from [Ciocoiu & Nau 2000, p4]

duction rules. But this deductive closure is not extensive enough, because of the implicit background knowledge of K (exformation) which should be made explicit by a formal ontology. This ontology is stated in a formal language; consequently, the sentences of S' can be interpreted further in L_0 analogously to the above example. Hence, one can derive a theory in L_0 which contains the declarative content of the sentences S and the explicated background knowledge. This theory T can now be mapped via the semantics of L_0 to a model which then can be restricted to a sub-model that excludes constructs that not explicable by K's or L's models, respectively. Finally, one takes this sub-model as the semantic model of the sentences S of K.

Obviously, the crucial part is the explication of the background knowledge via the formal ontology. This procedure reminds of communication's task of ex- and inzitation which relied on a pre-shared common conceptual space which is, in the case of formal languages, rendered as formal ontology containing a set of axiomatic sentences. Therefore the connection to the cognitive layer is hidden in the construction of such an ontology which is based on the ontology engineer's knowledge of the world, whereas the actual models are classical relational structures.

FOL and cognitive basis

3 Synopsis of Part I

The previous chapters introduced certain important interdependent theoretical constructs. The order of the chapters tried to demonstrate a possible trail through this broad field encompassing the fundamental notions used in the next parts of this work. Before entering the domain of pictorial representations in detail, the current results will be recapitulated and some areas will be mentioned which were beyond the scope of discussion due to the stringency of the exposition.

3.1 Cognition, Semiotics, and Conceptual Modelling

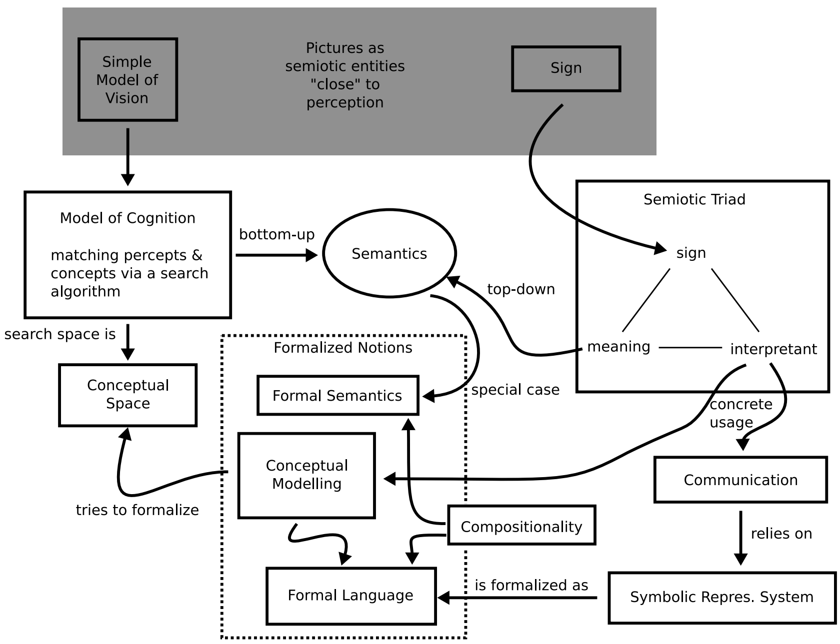


Fig. 3.1: Conceptual Map of Part I's important Waypoints

Fig. 3.1 shows the most important waypoints of the first part. The three large areas that were covered can be centred around semantics. First, a model of cognition was introduced in ch. 1 which described the mapping of percepts to concepts (sec. 1.4) via an algorithmic search (sec. 1.5) in the conceptual space (sec. 1.3). Second, some basic investigations of the semiotic triad (sec. 2.1) led to a more detailed, layered look onto communication (sec 2.2) as an exchange of (a) signs, (b) information, and (c) knowledge. These distinctions prepared the introduction of symbolic representation systems also known as languages. Following the path of scientific abstraction, a formalized counterpart was given to these notions making additional (meta-)remarks about the role of formal languages in this abstraction process itself (sec. 2.5, 2.6). All these efforts helped to introduce a framework for categorizing different approaches to semantics which was used intensively in sec. 2.4.

3.2 Notes ∗

Philosophical Preliminaries

These preliminary postulates are heavily influenced by the philosophical school of constructivism. This paradigm is only a marginal discipline in today's philosophical discourse but the only 'logically possible' consequence regarding this work's context: the influence of advances in (neuro-)biology and cognition, Nelson Goodman's foundation of the semiotic process in "The Ways of Worldmaking" [Goodman 1978], and new paradigms like "Cognitive Linguistics" and "Cognitive Semantics". A well written first approach to this philosophical field even for the sceptic is given in [Fischer 2000]; other works which influenced these postulates are [Pörksen 2002], [Maturana & Varela 1987], [Maturana 1978], [von Glasersfeld 1995], [Bateson 1979]. An important difference is postulate 2 which relativizes constructivism's inherent denial of the (knowledge of the) existence of an external world and therefore allows to embed even objectivistic positions as subcases, e.g., the inclusion of the [-constr.] semantic approach into the cognitive constructivistic notion in sec. 2.4. This branch of constructivism is known under the term constructivistic realism.

Cognition

The cognitive approach is heavily influenced by Gärdenfors's ideas and the modelling of the percept-concept matching as search algorithm originated from the computer science view of his book [Gärdenfors 2000]. This algorithmic view onto cognition would maybe allow to use the complexity measures of computer science to this method of bridging the gap. Nevertheless, and as stated multiple times before, this model is not based on neuro-biological evidence and it does not explain how we, as humans, apply this pattern matching but it is only a solution designed to explain how to connect symbolic patterns to a cognitive origin.

Another interesting approach towards perception systems and their relation to symbol systems is given in [Barsalou 1999] [Barsalou et al. 2003]. Barsalou introduces perceptual symbols as modal and analogical in difference to (formal) language's arbitrary, amodal, and non-perceptional symbols. These symbols are part of a larger framework including simulators and frames which allows complex reasoning tasks. In comparison to the given approach, Barsalou's argumentation focusses on the interplay of perceptual symbols and symbols of a language whereas the connection between real life objects and perceptual symbols is only discussed marginally. However, this thesis will take advantage of his formalization of reasoning on perceptual symbols in part II.

A totally different theory is proposed by Harald Atmanspacher and Peter beim Graben in [Atmanspacher & beim Graben 2006]: they describe the emergence of mental states from neural states by partitioning the neural phase space in terms of symbolic dynamics. This approach would include a neuro-biological point of view but presupposes a highly sophisticated level of mathematics.

Imagery Debate

The imagery debate asks the question whether the basic entities of cognition, the Mms (def. 3), have either the character of pictures, are solely of propositional nature, or have any other implementation, cf. [Gottschling 2003], [Weidenmann 1988], and [Arnheim 1988]. Avoiding any definite answer to this question, the previous chapters at least assumed a strong perceptional tie between mental imagery and a perception-based origin.

Semiotics

The given short cut through semiotics avoids certain well known principles and categorizations of this area like the classical distinction of syntax, semantics, and pragmatics because, in the context of pictorial representation systems, a separation of syntactic and semantic features is nearly impossible.[1] As Jon Awbrey stated on the SUO-mailinglist (`http://grouper.ieee.org/groups/suo/email/msg02595.html`), Peirce never used this distinction which was popularized by Morris and held its position in linguistics since then. Another important part of Peirce's work left out is his triadic ontology, which will enter the analyses of the next part via the backdoor of the sign classes index, icon, and symbol. An interesting view onto Peirce in the light of system theory's constructivism is given in [Scheibmayr 2004]. There, Peirce's semiotic theory is compared and incorporated into Luhmann's social system theory [Luhmann 1987] which shares some of our our philosophical preliminaries.

[1] hence, when approach semiotics from pictures, the leitmotif of grammar changes to "semantics precedes syntax" (p83);

The works of Paul Grice would propose additional starting points into semiotics from a pragmatic entry [sep:grice], e.g.,[Grice 1989].

Semantology

Gehring and Wille introduced the neologism *semantology* [Gehring & Wille 2006] to represent the theory of semantic structures and their connections, especially regarding scientific languages (a distinction based on Peirce's classification of sciences [Peirce 1992]) and their formal language semantics. According to the focus on knowledge representation and the interplay of formal language semantics, formal ontology, and knowledge engineering's intended (real-world) semantics, this thesis can be regarded as a contribution to the theory of semantology.

Meaning

As stated in [Materna 2004], there is a terminological chaos with the semantic concepts "sense", "meaning", "denotation", and "reference"[2]. This work strictly used the term "meaning" to describe the semantic labelling relation's relatum$^{\rightarrow}$ to which a semiotic entity is labelled to. The other words only make sense in the context of a semantic theory, e.g., sense (Sinn) and denotation (Bedeutung) are typical Fregean terms [Frege 1892], reference plays an important role in Goodman's work [Goodman 1968], and Carnap talks about intensions and extensions [Carnap 1988]. A comparison of these terms would be another practical example of the proposed framework's usability which lies beyond the scope of this work. A good introduction of the basic pillars of a philosophical approach towards semantics is given in [Coffa 1991] which covers the most important theories before the rise of psychology in the 20th century.

[2] the discussion in the next part would add "depict";

Physical Symbol Systems

Another approach to symbol systems is given by [Newell & Simon 1976] which introduces physical symbol systems as "*computer sciences' most basic law of quality structure*". Symbols are based on physical patterns which allow instantiation (type-token idea) and the formation of expressions. The *Physical Symbol System Hypothesis* states these systems as being sufficient and necessary for general intelligent action [Newell & Simon 1976, p116]. From the point of view of chapter 2 which combined Goodman's symbol systems and a perceptional semantic basis, Newell and Simon's symbols can be regarded as a simpler approach which emphasizes the importance of symbol systems to computer science. Nevertheless, the property of being *physical* can also be applied to symbol systems as postulates 2 and 3 allow to ground at least some – if not all – symbols in the physical reality of an object world.

Physical Symbol System Hypothesis

Formal Ontology

The above mentioning of formal ontology bypassed the classical ways of introducing this fields like [Sowa 2000]. Therefore formal ontology's origin in a certain view onto logic coupled to a philosophical theory of ontology[3] gets out of focus. Furthermore, formal ontology becomes a formalized notion of the concept of exformation which is introduced in the context of communication; in the light of Gruber's definition (ontology as conceptualization), it can easily be embedded into this part's antecedent, classical discussions of semantic foundation and therefore looses its buzzword aura.

[3] see [sep:logic-ontology] for an overview of different possibilities to combine these fields;

Part II

Diagrammatic Representation Systems

A Conceptual Modelling Approach

Focussing Diagrams

After the previous chapter's considerations, this part will resume the discussion of pictorial presentations. The next chapters will try to propose a conceptual model of the domain of «diagrams» with special regard to its derivation from the more general term «picture».

domain of pictures / diagrams

First of all, having stated the goals of the entire enterprise, a general introduction of «pictures» is inescapable. This results in the presentation of a meta-model that allows to classify different modelling approaches to the picture domain. With the help of a simple modelling recipe (see appendix F), the next step will present an FCA based modelling attempt. To improve the shortcomings of this approach, a simple axiomatic model of «abstract diagrams» will be introduced which will include the diagrams that are applied in conceptual modelling. Consequently, the idea of formal languages will be transferred to diagrammatic symbol systems.

With the help of a categorical view onto perception, conceptual representation, and semiotic representation, a comprehensive model of the entire domain of diagrammatic representations appears possible.

Before the next part presents conceptual graphs as a diagrammatic formal language whilst accentuating their problematic semantic and ontological foundation, an additional view from the GFO perspective onto diagrams will round off this part.

4 General Remarks on Approaching the Domain of Diagrams

Before developing a conceptualization, the next sections will elaborate the research questions of sect. 0.2 and motivate the conceptual modelling methodology of this part's remaining chapters.

4.1 Initial Situation: Diagrams and Conceptual Modelling

Conceptual modelling, as it was introduced in sect. 2.5, depends on a formal language framework. Thus, this underlying framework influences and restricts the creative act of modelling itself by confining the spectrum of expressions and by proposing several standard procedures and solutions. Ere stating a model in a formal language, the modelling engineer approaches the domain[1] in an intuitive way in which diagrams play a central role.

[1] regarding def. 2.11: the domain to model is in the majority of cases an excerpt of reality;

Basic Hypothesis 7

sketching a model

> Conceptual modelling begins without explicit premises, or at least should. At first, the domain is grasped in an intuitive way and most often by diagrams or pictorial presentations of prototypes (cf. appendix F). These simple diagrams often lack a formal language background, but are transformed more easily into a *diagrammatic* formal modelling language than into other forms of (sentential) knowledge representation.

The fact that diagrams can be considered as the external representations of internal mental images [Weidenmann 1988] could support the importance of diagrams in modelling even further, but this link will be left out in the following discussion to avoid the imagery debate [Gottschling 2003][2]. Nevertheless, the strong tie between pictorial presentations and the internal conceptualizations that heavily depend on perception (cf. chapter 1) will prove to be an appropriate cognitive underpinning for the above hypothesis.

[2] see notes chapter of part I at p52; this simple link would form a substantial thread of argumentation in the following;

4.2 Outline of the Intentions behind this Approach

On the one side, these conceptual modelling diagrams form an important group of formal KR-languages in practice; on the other side, their field of study lacks the research tradition of linear, sentential language and logic.

Retaining the idea of the pictorial turn (sect. 0.1), the following discussion will carry on part I's investigations of the interplay between formal semantics and real-world semantics towards diagrammatic representations whereas the approach will start from the more general concept «pictures». Analogously, it will culminate in the same dilemma: the discrepancy between formal semantics and the meaning that originates directly in the domain. However, to undertake this investigation, the domain of diagrams needs to be modelled first itself; subsequently, the general discussion of semantics must be revised and extended to include diagrams and their "closeness" to perception.

4.3 Starting Points into the Picture Domain

As the field of research of this thesis is profoundly transdiciplinary[3], a vast range of starting points can be taken into consideration. As modelling never starts from scratch but is based on previous results and an ubiquitous common sense approach, these underlying *pre-conceptualizations* (or pre-categorizations), i.e., pre-existing conceptual models, have to be made explicit.

[3] transdisciplinarity as a special form of [wp:Interdisciplinarity]; viz [Newell 2001];

pre-conceptualizations

Since most other attempts to the picture domain focus on another level of generality or utilize another granularity of formalization, the following approach has to be regarded as sui generis. Nonetheless, the presented results can be compared, at least partially, to already existing research results.

sui generis

The next paragraphs will introduce three possible entries into the diagram domain which would seem appropriate from a computer science point of view. As the proposed method of modelling will demand carving out the underlying pre-conceptualizations, a survey of other important investigations and their results will be given later in sect. 6.2.

Database Approaches

Storing pictures and their appropriate meta-data in databases requires an underlying model which focuses mainly on physical properties (for example: resolution, size, production date, etc.) as well as on simple semantic descriptions using subject catalogues or short natural language descriptions. There are several competing object-

picture as "physical" object

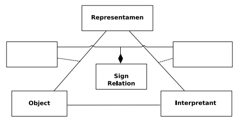
oriented models, e.g., the Object Oriented Image Model of Peter Stanchev [Stanchev 1999] or the Dicom standard[4] which plays an important role for medical imaging.

From the point of view of computer science, object-oriented models that are utilized in image software would be the most natural starting point which would additionally include already rigorously formalized object-oriented models. Nevertheless, these models often lack a theoretical background in contrast to the following approaches.

[4] cf. Dicom working group `http://medical.nema.org/` ;

Symbolic Representation

Regarding the strong connection of diagrams to semiotics, a formal model of symbolic representation on the basis of part I of this thesis, e.g., a formalization of Peirce's meaning triangle (cf. 2.1), could be another entrée. Besides the vast amount of different proposed approaches to semiotics[5], pictures and diagrams can not be classified as sole semiotic entities; further, the question whether semiotics is applicable at all still remains open [Sachs-Hombach 1998a] [Gerhardens 1998]. Nevertheless, basic terms of semiotics, e.g., the distinction between syntax, semantics, and pragmatics, will help to categorize different points of view onto pictorial representations; these will play an important role in the meta-model of sect. 5.3.



(fig. 2.1)

[5] cf. section on semiotics and the corresponding notes as well as ch. 2 and sect. 3.2;

Image Recognition

The research area of image recognition would propose an approach towards the picture domain which shares the basic ideas of the symbolic entrée but with a focus on an implementation of the underlying cognitive basics with the help of algorithms. This research area combines ideas from computer science (e.g., pattern matching as machine learning) with results from cognitive psychology and empirical research. Albeit lacking a thorough formalization of the background assumptions, results from this research area would be the starting point for the empirical research of this works basic hypotheses. For example, the controversy about Geon theory [Biederman 1987] [Tarr et al. 1998] would be a possible first step towards the basic building blocks of diagrams whose existence will be postulated later in section 7.1.1 ("graphiques").

Bildwissenschaft

The following discussion of diagrams leads to and is embedded into the research area of Bildwissenschaft, which is the appropriate transdisciplinary approach towards pictures. There are several attempts to model the domain of pictorial presentations or at least sub-categorizations regarding certain properties which often lack the formal stringency needed in the following. However, the basic axiomatics of [Sachs-Hombach 2006] will be the starting point of the following discussion.

5 Pictures and Diagrams: Basic Axiomatization and a Meta-model for Approaching the Domain of Pictorial Presentations

The pictorial turn demands to enter the discussion of diagrams from the general concept of «pictures». Hence, a model of pictures is required which will be used as the basis that underlies the approach towards a categorization of diagrams.

First, pictures will be differentiated from non-pictorial entities. Then, based on this basic agreement, which will be given by axioms only, these pictures are approached by a general meta-model entrée which formalizes the different points of view onto a single picture with the help of layers.

5.1 General Approach towards Pictures

As diagrams can be subsumed under the general concept «picture», a basic axiomatics based on the introductory literature of Bildwissenschaft is presented before entering the field of diagrams in the next chapter. A survey of introductory literature to Bildwissenschaft will be given in this part's notes section (sect. 10.1).

Retrospecting part I's original starting point to perception and semiotics, the most basic axiom was given already (p8):

Main Axiom 1 — pictures

> Pictures are signs that are close to perception ["wahrnehmungsnah"].
>
> [Sachs-Hombach 2006, p74] (own transl.)

pictures: between perceptional resemblance and symbols

Hence, this axiom connects the analysis of pictures with a theory of cognition (cf. sect. 1.5) and therefore resemblance (def. 1.6); further, it embeds these pictorial signs (sect. 2.1) into the context of (physical) symbol systems (def. 2.6 and notes section 3.2). Pictures are both close to perception as well as semiotic entities, however, they are neither one of both extremes only. Thus, the nature of «being a picture» resides in a special kind of interconnecting these extrema.

Because axiom 1 is too shallow to decide the subsumption of an object under «picture», the following characterization presents a catalogue of properties that emphasize a certain understanding of pictures: pictures in a strong sense[1] exclude natural pictures like reflections, as well as pictures that do not participate in communication, i.e., without at least an underlying intension of a sender to transfer meaning, e.g., a certain fact.

[1] see [Sachs-Hombach 2006, 74ff] for a detailed discussion and comparison to a broader notion;

Postulate 8

pictures (strong sense)

> Pictures are artificial, plane, and relatively lasting objects that are used as a part of communicative acts to illustrate real or fictitious facts.
>
> [Sachs-Hombach 2006, p77](own transl.)

pic vs. linear text

Pictures can easily be distinguished from linear text, as the latter is neither close to perception, because resemblance does not play a role in culturally determined semiotic denotation, nor plane, i.e., diagrammatic.[2]

pic vs. image

An important distinction resides between pictures and images. This distinction can be traced back to the difference between εικόν (Greek: 'eikon') and imago (Latin), which both referred to the most basal results of perception [Scholz 1991, introductory chapter]; in the following, the term image will always be used as mental image (def. 3) whereas pictures are coupled via cognition and perception to an external (physical) object – the *carrier of the picture*. As already mentioned, mental imagery will be left out of the discussion as far as possible to avoid the imagery debate.

carrier of the picture

To conclude, there are several points of view onto a certain picture which all are intertwined tightly but must be untangled when perceiving a picture and mapping the resulting percept to a concept. The following example introduces a picture as a compound of objects according to the dissection of the upper characterization.

[2] but one cannot draw clear borders: sect. 1.6 introduced ideographic and pictographic languages which both originate in resemblance and even simple enumerations leave linearity behind to express item's similarities by diagrammatic opposition;

Example 9 : Obtaining the Meaning of a Picture

> The original pictorial representation of fig. 5.1 is composed of a physical object – its carrier, which has certain material properties. The cognitive conceptual search algorithm (sect. 1.5) recognizes two distinct categories: patterns that resemble a sub-concept directly, and patterns that are characters of a semiotic system; these two are to be handled differently: whereas both are mapped to a corresponding Gestalt-pattern, semiotic entites are translated to concepts by semantic labelling rules in the background knowledge contrary to the search for a composite concept that matches the structure of the Gestalt-patterns.[3]

[3] cf. discussion about perceiving pics vs. text at p18;

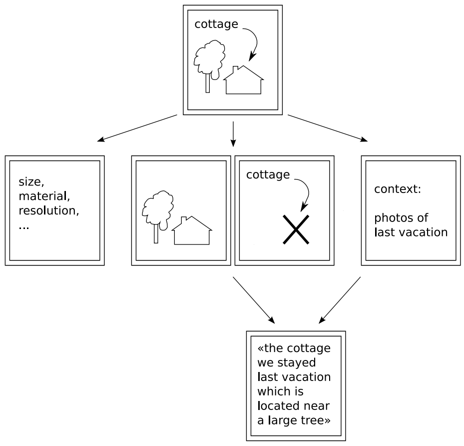


Fig. 5.1: Decomposing Pictures into Layers

In the above example picture, a house and a tree pattern are separated from the sign of an arrow whose source is given by a word pattern. As pictures take part in a communicative act, the background knowledge contains additional information about the co- and context[4]; in this example, the context is the situation of showing last holiday's pictures. From this information, the cognitive algorithm can derive *one of the different possible* meanings of the picture: «last holiday's housing».

[4] see the later discussion of heterogeneity (p73) that will introduce both terms;

Taking a view from the meta perspective on the previous example, one can differentiate different views onto a picture which are interrelated and of which the semantic content can be regarded as the most complex composite.

5.2 Küker's Different Ways of Viewing ∗

There are other ways that lead to different viewpoints onto pictures. Andreas Küker's approach, that introduces a philosophically coined view onto the debate, will be sketched briefly [Küker 2007]. First, Küker introduced three different ways to look at pictures: an ephemeral, quick first glance, an aesthetic reception, and a theoretical examination. The whole examination that is presented in the surrounding chapters can be considered as a theoretical analysis of pictures, whereas the notions of a first glance and aesthetics are meta-descriptions of the viewer himself which result in different modes of cognition.

look at pics.: ephemeral vs. aesthetic vs. theoretical

The ephemeral view will be confronted with the theories of Weidenmann and Schnotz when describing the different ways of reading pictures (p83) whereas the aesthetic quality of the ephemeral view will be left out of the discussion as well as the entire field of aesthetics which would imply a basic analysis of (aesthetic) qualities beyond the scope of this thesis.

Second, Küker reduced pictures to *transformations*, i.e., a process of uncovering and visualizing; thus, the most important feature of a picture is not its denotation but the interrelation of its different aspects. Consequently, communicating with pictures is a form of explicating and hence related to the approaches that will later be classified as being pragmatic. Further, the idea of pictures as a transformation becomes manifest in the categorical approach of chapter 8 and will turn up again in the discussion of Tufte and Bertin's approach towards diagrams (sect. 6.2.3 and 6.2.4).

transformations

5.3 A Layered Approach to Modelling the Picture Domain

Based on the primordial distinction between different views onto a picture, as proposed in the previous example, and the theoretical results of part I, different points of view onto pictures can be categorized by an abstract (meta-)model. This meta-model would be the foundation for any comparison of different conceptualizations of the picture domain which will be introduced in the following. In addition, fig. 5.2 visually grounds the entire model onto the cognitive model of chapter 1 and chapter 2's semiotic framework which were shown to be the heart of the underlying approaching of pictorial representation.

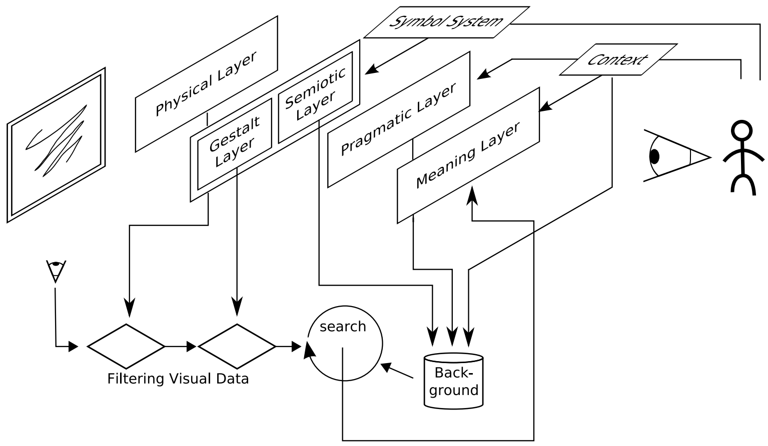
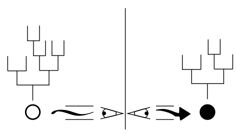


Fig. 5.2:
A Layered Approach to Picture Vision and the Role of Concept Recognition

Similarly to the two ways of approaching a communicative act which were proposed by Eco (fig. 2.4), the above diagram has two points of view: the recipient sees the picture through a stack of different layers which depend on each other sequentially due to their embedding in the cognitive search for concepts whereas the meaning of the picture is the most elaborated aggregate; when authoring a picture, the author wants to transmit a given meaning in a certain context and chooses the symbol system and the physical media in correspondence with one's own pragmatic intentions.



(fig. 2.4)

Two external entities include the recipient or author's influence on these actions: a context which describes the communicative situation and formalizes the background knowledge in which the semantic content of the picture will be embedded, as well as a (formal) definition of the used language system[5] that describes grammar and the semantic labelling function. The carrier of a picture influences the first filtering step of cognition which tries to compensate flaws of the visual sensor. As already explicated, Gestalt pattern matching dominates the conceptual search algorithm's preprocessing whereas symbols, as a part of a language, force the direct translation to concepts via the semantic labelling relation that is included in the background knowledge. Since textual representation plays an important role in abstract logical diagrams[6], a distinctive feature of diagrammatic semantics is the combination of Gestalt and symbolic pattern matching.

[5] this can also be seen as a part of a larger context, but is modelled as an explicit entity as it heavily influences the pattern decomposition and matching at the Gestalt-semantics layer;

[6] cf. heterogeneity type (f) of later discussion (p73);

The next chapter will attempt to model the domain of diagrammatic representation with a meta-view from the layered model above. Further, the layered model circumscribes the extent that an all-embracing model of the domain would have to cover.

6 A First Attempt to Model the Domain

The conceptual modelling approach dissects the domain of diagrams into classes of entities, their properties, and their relations. Fca seems an appropriate candidate to express the outcome of this process with the help of an Fca context that describes how properties (as Fca-attributes) inhere in objects to constitute concepts.

Appendix F combines the ideas of hypothesis 7 with a formal foundation in Fca. The modelling recipe describes the generation of a sketch of the domain with the help of simple rules of a (card-)game whereby the outcome is isomorph to Fca-contexts. This procedure derives concepts with the help of prototypes (which result in Fca-objects) and their properties (Fca-attributes).

The following two sections will introduce the set of prototypes together with a common sense categorization. Further, they will summarize the most important research results, i.e., pre-categorizations, that are the fundament for the game's properties.

6.1 Prototypes

The modelling game demands a representation of the objects of the domain; they will be given with the help of prototypical pictures. As the choice of these objects belongs to the modelling engineer and modelling is based on his experience, this process is influenced by pre-conceptualizations and a first idea about a common sense classification. Consequently, the subsequent list of pictorial objects will subliminally sketch important aspects of this underlying understanding, i.e., will give a description of the common sense understanding of diagrams by grouping the prototypes.

For example, the next two prototypes originate in the common understanding of a picture as an entity that resembles the depicted object as close as possible. Both 'fool' human vision by simulating the perception of the real object (see chapter 1: "closeness").

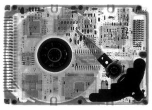
Simple print of a snapshot recorded by a camera on light-sensitive material [wn:photo]; photo

original picture found at `http://www.albverein.de/pliezhausen/`

(realistic) painting
abstract painting

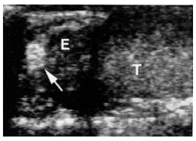
A *(realistic) painting* of the scenery already depicted by the above photograph; this is opposed to an *abstract painting* that does not reproduce real world objects in a way close to their original perception (cf. discussion of abstract art at p81);

With the help of imaging methods, other spectra can be made visible to human vision.



radiograph

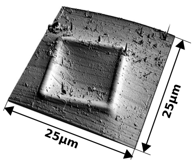
Photography beyond the light spectrum that can be recognized by human vision, e.g., a *radiograph*, but in an order of magnitude that is graspable by humans;
an example is this X-ray radiograph of a harddisk drive, found at
`www.matsci.ucdavis.edu/.../EMS-162L/EMS-162L.htm`



sonogram

A *sonogram*, i.e., an image of a structure that is produced by ultrasonography (reflections of high-frequency sound waves); a sonogram is used to observe foetal growth or to study bodily organs [wn:sonogram];
the figure shows a Papillary cystadenoma of the epididymis due to
`http://www.cc.nih.gov/ccc/papers/vonhip/papillary.html`

Further, other magnitudes or scales of granularity can be mapped to representations that are sensible to human vision such that the depiction retains the structures (homomorphy$^{\rightarrow}$) and the underlying measure.



AFM

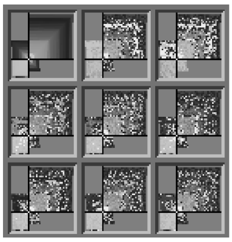
Data visualization of the spatial scan of a magnetic probe with an AFM ([wp:Atomic force microscope]) which measures the interaction between the surface of a sample and a scanning tip; the resulting measurement has a resolution between fractions of and (as in the case of the left diagram) tens of a nanometer[1];
[Petriconi 2006, fig. 8.1, p94]

[1] to compare: one atom has a diameter between $0.05nm$ and $0.52nm$;

An AFM picture originates causally from a measurement of a real entity; regarding the outcomes of the measurement, one can classify it as the simple representation of a test series, i.e., a collection of data. The following diagrams depict data in general.

chart

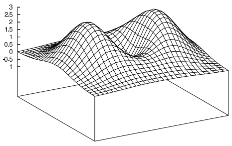
A *chart*, e.g., circle diagram or line diagram, depicting low dimensional data values; on the left, a simple exploded pie chart is depicted;



2 D projection of a large quantity of multidimensional data; the left picture was generated by the VisDB-framework representing 1000 of 8 D data objects [Keim & Kriegel 1994, fig. 6b], the original diagram which heavily depends on the usage of colour can be found in appendix A; see `http://www.dbs.informatik.uni-muenchen.de/dbs/projekt/visdb/visdb.html`

multidimensional data representation

The underlying data can be abstracted further to a simple mathematical description. This models the origin of the data as the outcome ("image") of a function and leads to the class of function plots. Hence, these are not connected to a causal, real world origin of the data but depict a functional relation only. Nevertheless, their visual style resembles the previous AFM diagram.

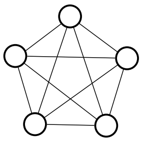


A *function plot*, i.e., 3 D plot of a mathematical function; GNUPLOT[2] generated representation of the function given by the formula $(x^2 + 3y^2)e^{(1-(x^2+y^2))}$

function plot

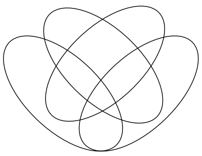
[2] `http://gnuplot.sourceforge.net`

Functional relations can be abstracted even further to relations between abstract mathematical concepts like graphs or sets.



Graph representation, i.e., a 2 D rendering of a mathematical graph→ ; here, a K_5, a complete→ graph with 5 vertices, is depicted;

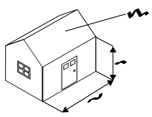
graph representation



Venn diagrams depict the relations between abstract sets with the help of the visual metaphors of cutting and joining; the diagram on the left shows a Venn diagram of four sets, viz all possible combinations of these;
the given arrangement is inspired by
`http://www.combinatorics.org/Surveys/ds5/VennGraphEJC.html`

Venn diagram

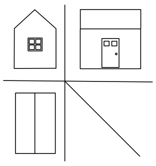
The following diagrams return to representations of real world entities but are embedded in a certain practical context: engineering.



3 D *engineering drawing* (2.5 D[3] projection of a 3 D object) including measure and annotations;

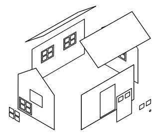
engineering drawing

[3] cf. p18 and the usage of stereopsis and perspecitve;



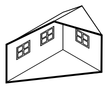
An *orthographic projection* (“Dreitafelbild” in German) is the simple, but ambiguous projection of a 3 D object into 2 D;

orthographic projection



An *exploded diagram* of the same object which allows to show the compositional parts and their arrangement with additional information;

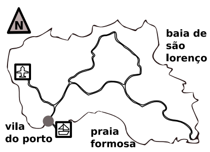
exploded diagram



A simple *cross section* cutting the 3 D object with a plane which allows to peek inside the solid object;

cross section

Another large group of diagrams are maps which exhibit certain degrees of abstraction starting from an one-to-one representation of a region of (real) space.

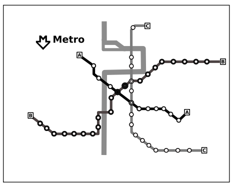


A *(road) map* depicting a region of space whereas the original geometrical measure is preserved and objects beyond the maps’ scope are added as well as abstract text labels and symbols (pictograms);

map of Azores Santa Maria Island due to

`http://www.azores.com/azores/santa_maria.php`

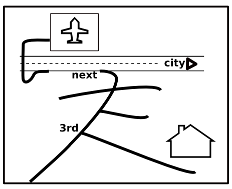
(road) map



A *subway map* only preserves topological properties whereas geometrical ones are often distorted;

map of Prague’s metro due to [wp:Prague metro]

subway map

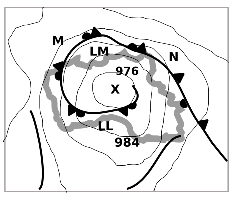


A *route sketch* simplifies the geometrical description of a region even further, sorts out “unnecessary”[4] information, and adds additional information by icons; further, it depicts a projection of a movement process onto space;

route sketch

[4] unnecessary with regard to a concrete pragmatic context;

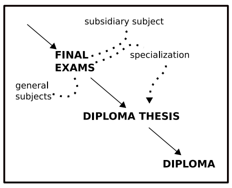
Hybrid diagrams combine two different styles of representation to aid one another.



Weather charts combine a stylized geographic map with conventionalized symbols depicting measurements of temperature and air pressure;

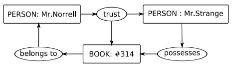
weather charts

One can also use the visual metaphor of a map to depict a “landscape” of abstract entities.



A *virtual map* uses the metaphor of a map to represent abstract entities and their relations; examples are [wp:Concept maps], [wp:Mind maps], and [wp:Topic maps];

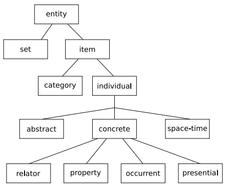
virtual map



Semantic networks (semantic nets), e.g., conceptual graphs, enhance topic or concept maps with additional symbols that depict abstract, multidimensional relationships, and often a well-defined (formal) semantics; the basic publications on semantic nets are [Brachman 1977], [Brachman 1985], [Allen & Frisch 1982], and [Woods 1975];

fig. 13.2, p130

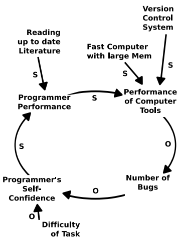
semantic nets



Instead of additional symbols, *order diagrams* use the spatial arrangement of entities to depict an order relation between them, e.g., a taxonomy’s tree-like hierarchy; another possibility to represent tree-like orders without the shape of a tree are treemaps as shown in fig. A.2 for the left tree graph;

fig. D.2, p170

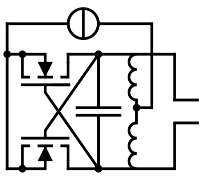
order diagrams



Representing abstract entities and their causal connection in time by *causal loop diagrams* introduces the importance of “arrow”-symbols and enhances spatial representation to time;

causal loop diagrams

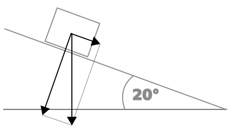
Besides the above classical diagrams, there are many specialized forms of representation used in science, education, and arts.



Circuit diagrams are symbolic representations of electronic devices and their connections in a human readable and traceable manner; nevertheless, one could assemble the devices in the depicted fashion to get a functioning appliance;

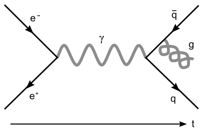
adapted from [wp:Electronic oscillator]

circuit diagram



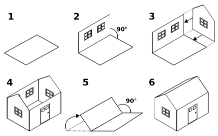
In the context of classical mechanics, *free body diagrams* help to visualize the spatial position of stylized objects and the forces operating on them;

free body diagrams



Leaving the scope of human perception, *Feynman diagrams* depict the quantum mechanic relations of particles;
due to [wp:Feynman diagram]

Feynman diagrams



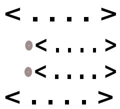
An *assembly diagram* shows a process by consecutive snapshots, thus depicting a temporal series; analogously, *story boards* and comics depict temporal relations of single sub-pictures;

assembly diagram
story boards

Finally, there are diagrams that can not be classified by the upper pre-categorization and will be used as special cases to measure the quality of the following conceptual models.

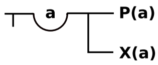
Randomly generated inkblots are the basis of *Rorschach inkblot pictures* [Rorschach 1921]; it is not clear whether these pictures fall under the definition of pictures in a strong sense (def. 8), as they originate in an "accidental" act of creation that is contrasted by the viewer's semantic interpretations which form the outcome of the psychological test;
from `http://ar.geocities.com/rorschach_inkblots/`

Rorschach inkblot pictures



The classical counterexample to diagrammatic representation is *linear text* which gradually includes diagrammatic elements such as lists or enumerations; a good overview of the importance and of the theory of lists is given by [Tufte 2006a];

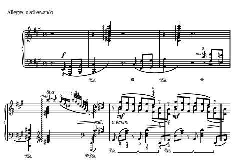
linear text



Frege's diagrammatic notation of first order formulae of the *Begriffsschrift* [Frege 1879] as well as Henkin's branching quantifier[5] are examples of non-linear logic formulae;
left: rendering of "Not all *a* that are *X* are *P*"

Begriffsschrift

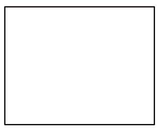
[5] `http://planetmath.org/encyclopedia/Branching.html`



sheet music combines symbolic notations with a temporal sequence and additional information with the help of a sophisticated notation system;
Arthur Gray's "Les Néréides" rendered with LilyPond[6]

sheet music

[6] `www.lilypond.org`

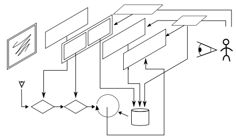


the most important picture (or diagram) is the empty picture (diagram) – the *tabula rasa*, which is an important topic in the history of arts [Wagner 2004]; further, a description of digrams based on a (graph-)grammar depends on these as the starting points;

tabula rasa

6.2 Summary of important (Pre-)Categorizations from Literature (∗)

The following section will introduce the main research results of Bildwissenschaft regarding the diagram domain from a pictorial point of view. The following presentation of pre[7]-conceptualizations will utilize the meta-model of fig. 5.2 to interrelate these different approaches and to group different theoretical attempts with respect to their covering of a (meta-)layer.



(fig. 5.2)

[7] "pre" in the sense of both before the approach presented in the following and pre-formalized as most were not presented with a rigorous formal foundation;

Additionally, an example will highlight the most important step of modelling in a prototypical way: the extraction of properties, which are related to FCA-attributes by the modelling recipe, from already existing models, or at least from the ideas behind these pre-conceptualizations.

Example 10 : Extracting Properties

> As the possible ways to extract FCA attributes from the following considerations are multitudinous, this example will concentrate on inferring one property which will be tracked through the steps of modelling. This will be the attribute based on the concept of «abstractness».
> (This example will be continued throughout the following sections.)

This section is marked (∗) as the following presentation outlines most results only abridged and without going into the necessary detailed discussion; further, it does not present the underlying philosophical background which often disagrees with the postulates of sect. 1.1. However, the presented findings will be the underpinning for both the FCA modelling and the two later approaches.

6.2.1 Approaching the Gestalt-Semiotics Layer

The layers of the meta-model can be grouped around the Gestalt-semiotic layer because the mapping to percepts is central to pictorial representation. As pictures are semiotic entities, properties of syntax and semantics[8] which were already introduced in chapter 2 can also inhere in pictures.

[8] whether both are applicable to pictures will be discussed from a meta-level standpoint later;

Regarding the following analyses, the most important aspects can be tracked back to formal language and its semantic foundation as well as to semiotic features of the underlying symbol system; for example, the classification of different types of heterogeneity, i.e., the usage of word labels as a part of the graphical language (detailed at p73ff) and the application of *control codes*, e.g., the numbered steps of an assembly diagram that give a hand to the reader (and consequently are connected to the pragmatic layer).

control codes

The Peircian Trisection: Icon – Index – Symbol

One of the most discussed topics in semiotics is the status of pictures regarding Peirce's[9] trinity of *index* [CP, 2.248], *icon* [CP, 2.247], and symbol [CP, 2.249] which is based on his basic ontological trichotomy into firstness, secondness, and thirdness. This trisection plays an important role in the classification of signs in classical semiotics.

[9] cf. notes about Peirce in sect. 3.2;

Following [Pape 2004], a simple, brief explanation of these categories can be given with the help of relational modelling: (a) firstness is the state of being unique and without relation to some other entity, thus it can be described with the help of an unary relation; (b) secondness is something that exists independently but which depends on another (unique) entity; the distinction between those two entities relies on (c) a third entity which manifests this separation.[10] Another view utlizes the following metaphors: (a) pure possibility of being, (b) an existence which always depends on at least one other entity, and (c) a rule capturing the thirdness' distinctive power. Regarding Peirce's semiotic triangle of sect. 2.1, these three categories correspond to the entities (a) respresentamen, (b) object, and (c) interpretant.

firstness, secondness, thirdness

[10] from a system theoretic point of view, this second entity results in the definition of a boundary (third entity) for the first one which is the basic reason for the first' being distinct;

When applying this basic ternary analysis to the trinity of the semiotic entities, one can distinguish at least nine different categories; the most important are: the secondness and thirdness of the sign itself which are known as *type* and *token*; and, regarding the relation of secondness between the object and the sign (labelling relation), the three classes (a) *icon* that exhibits statistical regularities, (b) *indices* that depend on usage by habit, and (c) *symbols* that solely depend on a labelling relation given by rules.

type & token

icon, index, symbol

To clear up these at a first glance elliptical definitions, these three entities will be contrasted to the basic model of perception: symbols are the entities which are solely mapped to concepts by background knowledge's rules, i.e., are altogether arbitrary and have to be learnt, like English words; icons directly resemble the depicted object in a statistical measure, e.g., a certain photo of a horse resembles the original horse up to 95%; and indices denote a certain functional relationship which is based on certain conventions and experience, e.g., the tracks of a horse which index a horse's passing by.

Several authors point out the unclear distinction between these three semiotic classes. For example, Schnotz and Eco's object[11] that even iconic signs are only conventional signs like symbolic ones [Eco 1976] [Scholz 1991]. Nevertheless, these three classes can be reduced to a basic distinction of perceived objects based on the way they are treated by the cognitive algorithm.

[11] "*the iconic signs reproduce some of the conditions of perception of an object, but only after they have been selected and explained on the basis of graphical conventions*" Eco due to [Strothotte & Strothotte 1997, p52];

Schnotz proposed to avoid the icon-symbol distinction in favour of the dualism of *intrinsic and extrinsic representation* [Schnotz 1993]. An intrinsic representation

intrinsic vs. extrinsic representation

depicts facts by two-dimensional, areal objects that are perceived *like* symbols but are of iconic origin.

In the following discussion, only the different handling of these signs by the underlying perceptive transformation will be focussed when analyzing the semantic layer.

Heterogeneity

heterogeneity

The property of *heterogeneity*, i.e., the utilization of both iconic symbols and text, can be extended to a classification of the gradual interplay of <....> and  [Hammer 1995].

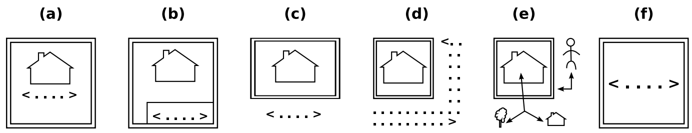


Fig. 6.1: Different Grades of Heterogeneity

In fig. 6.1, the steps from (a) to (d) explicate the translocation of a textual representation from the area of the picture to its environment: a text label can (a) tag a iconic entity directly; this labelling can be restricted (b) to a legend on the area of the picture; in the next step, the text leaves the picture as (c) a caption that accompanies the picture and represents its title, a summary, or additional information; a picture is embedded into its (d) cotext, i.e., the text that surrounds the picture, and its (e) context which is independent of the sourrounding textual or pictorial representations[12].

[12] this statement seems hypothetical here, but, due to the prominent role of background knowledge in the conceptual cognition algorithm of sect. 1.5, there can be no cognition without context;

The important case, e.g., for the later definition of abstract logical diagrams (see def. 7.4), is type (f): the usage of text, e.g., a word label, directly on the diagram's plane. In contrast to (a), this allows to depict abstract concepts that cannot be depicted with the help of an icon. This leads directly to different semiotic feature's impact on semantics.

6.2.2 Semantics

The layer of semantics cannot be detached from the underlying cognitive model as well as semantic labelling relations in the background knowledge. Consequently, the semantic layer cannot be approached exclusively.

Classifying the Depicted Object

When restricting a picture to the outcome of its perception, the classification of the denoted object can be transferred to the picture itself. This depends on an already existing categorization of the external world, e.g., a formal ontology like GFO. Regarding the previous prototypes, the following categories seem appropriate: (a) a picture depicts a situation[13] (e.g.: free body diagrams) or its projection onto (b) processes (causal loop diagrams) or (c) space (road maps). The temporal representation of processes allows for further differentiation, as the diagram can depict snapshots at different time points (assembly diagram) or the temporal sequence itself (classical flow-chart). Besides the depiction of a real-world space via geometric or topological homomorphy→, diagrams allow to depict imaginary spaces, e.g., vector spaces (function plot) or a metaphorical representation of the relations between abstract objects (like semantic nets).

situation vs. process vs. space

[13] in the terms of GFO (see appendix D) one would prefer situoids over situations; cf. discussion in sect. 9;

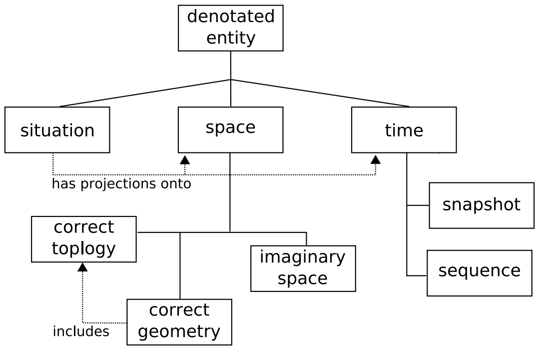


Fig. 6.2: Classification of the Depicted Object

[Schnotz 1993] differentiated between conceptual category structures, topological structures, and process abstractions, as well as hybrids. Recognizing these conceptual structures as imaginary spaces, his view can easily integrated in the above taxonomy.

Strothotte's Trisection: Presentational vs. Abstract Graphical Pictures vs. Pictograms

Christine and Thomas Strothotte proposed another dissection into basic pictorial entity types: (a) presentational pictures, (b) abstract graphical pictures, and (c) pictograms; but these classes are not disjoint as depicted in fig. 6.3. [Strothotte & Strothotte 1997, p44f].

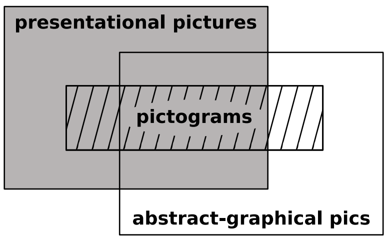


Fig. 6.3: Strothottes's Basic Classification [Strothotte & Strothotte 1997, fig. 3.9, p57]

Presentational Pictures "*present properties and relations in reality (including virtual reality and imagination) which are visible to humans. Although parts of reality can be distorted, manipulated, or otherwise misrepresented, geometric and physical*

presentational pics

aspects of surfaces and the behaviour of fluids or real objects are of central interest in presentational pictures" [Strothotte & Strothotte 1997, p44]. Hence, this class of pictures includes many different examples from photos to line drawings.

abstract-graphical pics

Properties and relations in reality which are *invisible* to human perception can be presented in an abstract graphical manner. *Abstract-graphical pictures* visualize these non-perceptional entities with the help of graphical symbols – Strothotte distinguishes geometric patterns, arrows and lines, and finally text labels –, and the mapping of invisible properties onto visible attributes.

pictograms

Finally, *pictograms* represent something more abstract than they actually depict, e.g., a photo of a horse that prototypically exemplifies the class of all horses in a dictionary[14]. The meaning of pictograms is either obvious to the reader, i.e., easily accessible by common sense, or has to be learnt. Therefore, pictograms are used analogous to (Peirce's) symbols: they are unambiguous, their meaning is well defined and, regarding a certain context, they can be replaced by words, i.e., translated to other signs. Hence, a pictogram's author aspires common understandability with the help of objects that are close to perception but utilized strictly symbolically or – with Schnotz – intrinsically.

[14] [Goodman 1968] calls this class of pictures "horse-pictures" and distinguishes it from "picture of a (certain) horse";

relation to icons and symbols

Including the Peircian distinction between icons and symbols, the interrelation between the three semiotic classes of the Strothottes (cf. 6.3) can be investigated further. Presentational pictures are dominated by iconic signs regarding certain conventions which are to be learnt in advance. On the contrary, abstract-graphical pictures are dominated by symbolic signs which are conventionalized regarding the context in which the picture is used; thus, these pictures always need a legend to explain the meaning of the symbolic sign to the reader. Consequently, by combining these two different approaches, pictograms can be considered to be pictures[15]. They "*can be defined by the context and the motivation of their use and not by dominant usage of one of the sign categories (in contrast to presentational and abstract-graphical pictures)*" [Strothotte & Strothotte 1997, p56]. As pictograms contain less information than most pictures, their meaning is easily remembered; therefore, they *seem* to be intuitively understandable. It is a common fallacy to state that they do not have to be learnt in advance (cf. Schnotz and Eco's objection to the Piercian classification of signs at p72).

[15] contrary to other authors like Horten [Horton 1994];

fallacy of intuitive reading

decision procedure

The difference between the presentational and the abstract graphical usage[16] of a picture manifests

in the dominance of graphical symbols: if abstract symbols dominate the picture, one tends to read it as abstract-graphical. Thus, the decision procedure has to ask: "Does the picture still make the intended sense without the graphical symbols?" [Strothotte & Strothotte 1997, p57].

[16] "*graphical symbols draw attention to themselves, because they often have to be interpreted via specialized conventions,...*" (see next page)

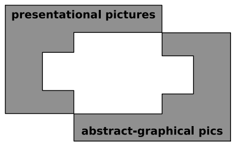
Regarding weather charts, this question has to be answered negatively as the central information is given by symbols, e.g., numbers representing temperature or air pressure; hence they can be classified as abstract-graphical pictures. Engineering drawings are a special case: above all, they are presentational but additionally abstract-graphical notions are applied, e.g., dimension labels. Whether this picture is classified either presentational or abstract-graphical depends on the purpose of the drawing: if the appearance of the drawn object is important in the concrete act of communication, it has to be classified as presentational.

[16] (continuation) "*. . . while understanding the iconic signs that build the presentational character of a picture is often automatic and unconscious*" [Strothotte & Strothotte 1997, p57];

Pictograms can be either presentational or abstract-graphical. In their role as a sign, they can be part of abstract-graphical pictures. If pictograms or abstract-graphical pictures are part of the reality to be depicted, they can be contained in presentational pictures[17]. Presentational pictures can become abstract-graphical pictures by adding graphical symbols [Strothotte & Strothotte 1997, p60].

[17] Strothotte uses the example of a CAD-program's screen-dump which presents the drawing's abstract-graphical features as a part of the depicted real-world;

As this classification depends on a pragmatic point of view, one can only differentiate pure abstract-graphical or entirely presentational pictures leaving out the hybrids (cf. 6.3).



(fig. 6.3 (excerpt))

Interlude: Semantic Foundation – from Pictures to Logical Diagrams

Reassuming the pictorial turn which focusses diagrams from the point of view of pictures, the semantic foundation of diagrams can be considered a special case of pictorial semantics. The following sections introduce an overview of basic approaches to this topic.

Following [Scholz 1991] and [Sachs-Hombach & Rehkämper 2000], the two classical semantic theories about pictorial presentation are based on similarity and causality. The foundation of the labelling relation of a picture onto the (visual) similarity of the depicted object to a real-world object leads to a variety of inconsistencies of which Nelson Goodman's mathematical objection seems the most fundamental: if a depiction of a horse is similar to a horse, then – with the mathematical definition of similarity as being reflexive – in the same way a horse has to share similarity with every of its possible depictions; regarding a simple abstract line drawing of this horse, this reflexivity seems inappropriate [Goodman 1968]. This objection was already addressed by the definition of resemblance (def.1.6), which avoided this shortcoming by introducing resemblance as a gradual quality that is based on the two notions of concept- and percept-equality. Both equalities are reflexive (a property which plays an important role for the mapping of percepts to concepts, but resemblance is only based on these reflexive notions without being reflexive by itself).

similarity approach

The causal explanation is based on the process of creating a picture. Regarding photography, the connection of the patterns which are made visible on the carrier via underlying physical and chemical processes appears to give a reasonable foundation of the denotation relation: the photography of a certain horse depicts the horse whose photo was taken to gain this picture. This approach could be easily extended to paintings and other imaging techniques (X-ray, sonography) but fails to explain the semantic link of fictional pictorial objects; for example, pictures of entities that are not existing in the real-word; such as, the picture of a unicorn (cf. the unicorn example in classical logic p34), or construction plans that are not carried out[18] but share a certain resemblance to the intended real objects.

causal approach

[18] "*It's part of an engineer, architect, and designer's daily business, to draw prospective objects*" [Scholz 1991, p37] (own transl.), therefore this kind of diagram is central to this work;

The following discussion will prefer a *cognitive approach* which is based on chapter 1. As already discussed, the approaches via causality and similarity are partially included in this model as they influence the mapping of percepts to concepts via background knowledge[19]; for example, perceiving a photography of a horse includes recognizing this object as a photo and thus "knowing" of the causal, physical process that underlies its generation[20].

cognitive approach

[19] another discussion supporting this decision is the indiffertiability of icons and symbols as argumented above because the idea of iconicity includes a basic measure of resemblance;

[20] this equals the perception of pictures as discussed in sect. 1.6: the percept representing the frame of a picture is a crucial input to the pattern matching algorithm;

Other Ways to Approach Pictorial Semantics ∗∗

The standard (philosophical) literature proposes certain other semantic approaches which will be outlined in a nutshell only (thus "∗∗"). Most of these are taken from or inspired by the overview in [Sachs-Hombach 1998b], [Sachs-Hombach & Rehkämper 2000], and [Sachs-Hombach 2001b]; they will be either embedded into the previous discussion or open a new field of argumentation themselves.

The restriction of (pre-)conceptualizations to the semantic layer is common to most philosophical approaches to pictures and diagrams of which Wittgenstein's was the first that focussed pictures anew in modern times' metaphysics.

Wittgenstein

As Thomas Hölscher emphasizes, there are different approaches to pictures in Wittgenstein's œuvre according to his two distinct creative periods [Hölscher 1998]. In his famous Tractatus which tried to establish a formal language foundation of the world[21] a picture is simply a statement about the world, a rendering of an object of the real world into a super-syntax of the Tractatus' formal language, or – as Wittgenstein states metaphorically – a mere window pane onto the world [Wittgenstein 1961]. These context-independent "über-pictures" are not restricted to two-dimensional, areal entities, but are part of the proposed formal language and can as such be analogously represented by both a diagram and a paragraph of text, i.e., a "sentence-as-picture".

Tractatus

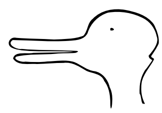
[21] this is harsh simplification, viz [sep:wittgenstein];

Wolfgang Stegmüller proposed a mathematically rigorous foundation of the concept «picture» in Wittgenstein's Tractatus with the help of Tarski's «relational systems» [Tarski 1941] (which were the basis of the relational structures of modern formal semantics) in [Stegmüller 1966].

Philosophical Investigations

Regarding the Philosophical Investigations' focus on language-games and the practical application of language [Wittgenstein 1958], pictures mature to self-contained entities which only testify facts about themselves but which can be related to external context by the practical usage in a language-game.

aspects



(fig. 1.8)

Ulrike Ritter extends Wittgenstein's basic pragmatic idea of pictures with his focus on aspects and the Goodmanian notion of exemplification [Ritter 1998]: labelling pictures to aspects extends exemplification to a context-dependent notion. For example, aspects can easily be applied to the duck-rabbit of example 2 (p17): with the help of structural resemblance, these two aspects can be recognized as either exemplifying a duck or a rabbit; focussing on only one aspect, e.g., the rabbit aspect, this picture can be read in an unambiguous way either strictly denotational, i.e., depicting a concrete rabbit, or as a fictive representation that is non-denotationally exemplifying rabbits in general. Hence, analyzing the way a certain aspect gets focussed can be related to choosing a certain starting point for the cognitive algorithm.

Pictures as Predicates

Playing around with the formal ideas of the Tractatus-approach leads to pictures as a representation of certain properties – in the words of formal logics: pictures are seen as predicates[22]. Hence, [Sachs-Hombach 2001a] connects the classical discussion about predication, which originates in [Frege 1879], to these picture-predicates. This discussion introduces the important role of a context to a concrete ascription of intensional properties to a picture.

[22] thus, they depict something like "sense" or intensions (p34f), as opposed to a strictly nominative usage;

Jörn Schirra [Schirra 2001] scrutinizes this connection even further. He emphasizes the dependence of a picture on its context, that is, due to Fauconnier, an entity of the mental space, and its consequences on the cotext of the communicative act in which it partakes [Fauconnier 1985]. He restricts the picture's predicative nature to *quasi-predicates*, i.e., signs that do not depend directly on any situation and that are neither utilized predicatively nor nominatively. These quasi-predicates instantaneously invoke (physical) reactions in the communicating agents; for example, the interjection "Fire!" is a quasi-predicate.

quasi-predicates

The idea of quasi-predicates is not applicable to all kinds of pictures because they are restricted to the usage in a language game: perceiving a picture is described as the addition of its semantic content to a situational context; a picture, in its role as quasi-predicate, allows to derive a new context of discourse. (Retrospectively, it will either depict a new fact about objects or a set of properties that are applied to the original situation, i.e., as either nominative or predicative.) Gaining the resulting new context incorporates a *harmonization* of the previous context's knowledge with the semantic content of the picture.

context harmonization

To conclude, Schirra defines pictures to be fictive, referential contexts which play the role of cotexts.

pic as context

A formalization of these basic ideas is the application of discourse representation theory (DRS) as a possible formalization of this dynamic semantics. DRS will be applied to semantically foundate conceptual graphs in sect. 12.3 (p124).

DRS

Deleuze

A different approach is taken by Michel Foucault, Gilles Deleuze and Félix Guattari. In the following the ideas of [Deleuze 1988] will be circumscribed briefly.

"*A diagram is a map, or rather several superimposed maps*" [Deleuze 1988, p88] whereas the «map» can be defined on top of abstract machines: "*a diagram or abstract machine [is] the map of relations between forces, a map of destiny, or intensity, which [...] acts as a non-unifying immanent cause which is coextensive with the whole social field. The abstract machine is like the cause of the concrete assemblages that execute its relations; and these relations take place 'not above' but within the very tissue of the assemblages they produce*" [Deleuze 1988, p37]

abstract machine

These quotes are not easily embeddable into this thesis' context because they form a fundamentally different entry to the picture domain and include a basically different view onto the world. The underlying machines are, following the introduction of Benjamin Kacas [Kacas 2003, p9ff], autopoietic processes that generate new principles of their own functioning[23], and, regarding the level of the imaginary and symbolic, develop different kinds of subjectivity. The notion of a machine resembles the ideas of diagrams as simulations, i.e., externalizations of mental derivations, as used later by Weidenmann. In a nutshell, Deleuze and Guattari approach language from a perspective that is totally different from chapter 2. Their semiotic theorization is based upon Peirce's semiotics, Hjelmslevs glossemantics, Prigogine's disequilibrium thermodynamics, and Batesons theory of strata [Kacas 2003, p10]. The central topics of this theory are the matter-form-difference, the dependence of codes on their implementation in different strata, and the interrelation to emergent processes. Nevertheless, this approach displays an inherent compatibility to the questions of chapter 1 and could result in new insight into semiotics.

The following discussion will only adopt a minor feature of these diagrams: processual diagrammatic relations, i.e., diagrammatic connections of the entities of a diagram, cannot be reduced solely to structural relations but bear an autopoietic meaning

[23] hence, resembling the ideas of [Atmanspacher & beim Graben 2006] or other system-theoretic or kybernetic approaches to semantics as mentioned in the notes of part I (p52);

[24] cited due to [Kacas 2003]; "*Le diagramme, en effet, est ici conçu comme une machine autopoïétique qui non seulment lui confèrer une consistance fonctionelle et une consistance matérielle, mais lui impoase aussi de déployer ses divers registres d'altérité, qui le font échapper à une identité fermée sur de simples rapport structuraux*" [Guattari 1991, p68f];

for the machine itself [Guattari 1991][24]. Regarding the discussion of the ontological status of a diagram's relations that will take place in sect.,13.3, this aspect seems to be left from the debate entirely as diagrammatic relations are considered to be representants of real world relations only.

Summarizing the Approaches to Pictorial Semantics

Recapitulating the previous paragraphs, there is no single theory that explains the semantic power of pictures and diagrams. The two major approaches, which are either based on similarity and resemblance or on causality, have counter-examples that cannot be denied. Nevertheless, both are able to explain the connection between the semantic content of a picture and real world entities. Regarding the cognitive model of ch. 1, perceiving a certain pictorial object as being a photo enters information about the causal origin into the background knowledge of the search algorithm which allows to draw a direct connection based on resemblance between the content of the photo () and the original source of the photo ().

background knowledge

The pragmatic embedding of a picture in a communication or its origin in a creative act allows to add other aspects to the background knowledge of the search algorithm for the meaning of a picture, e.g., pictures as predicates or simulation devices; but these belong to the pragmatic layer.

pragmatic determination of semantics

Compared to linear language semantics, the closeness of pictures to perception and the inherent connection to the underlying principle of conceptualization allow for more possibilities to influence the matching algorithm by background knowledge. If one restricts pictures to mere symbolic representation, they become nothing more than symbols of an ordinary linear language. Hence, linear symbolic representation can be understood as a special case of pictorial representation.

linear, symbolic representation as special case of pictorial repr.

Nevertheless, the essence of being a picture (or being used as a picture) resides in the cognitive closeness and, hence, the gradual dependence on resemblance as an important semantic feature.

The Fallacy of Abstraction

A central measure for the resemblance of a picture to some fundament in the real world is abstractness. This property was already included in the differentiation between representational and abstract-graphical pictures (p74) but seems to be a faux-pass word without an underlying formal definition. From an aesthetically coined view which is important in art theory and history of art, abstractness seems also an appropriate artistic measure. Further, pictorial abstractness can be grounded on the mapping to abstract concepts.

Abstract Art

The usage of «abstract» in art is best described by the art-theorist and (abstract-)artist Paul Klee: "*being an abstract painter does not imply to abstract from the resemblance to natural objects, but is based – independently of these possibilities of comparison – on the extraction of pure, visual relations*" [Klee 1964][25]. Thus, the two important feature of this artistic school are (a) the independence of the depicted relations from real-world objects that could be determined by any kind of resemblance, and (b) the expression of pure relations with the help of visual dichotomies like bright–dark, above–below, circle–square, e.g., the colour coding of the relational pair «back» and «forth» by the colours yellow and blue [Klee 1964].

[25] own transl. of "*als Maler abstrakt sein, heißt nicht etwa Abstrahieren von natürlichen, gegenständlichen Vergleichsmöglichkeiten, sondern beruht, von diesen Vergleichsmöglichkeiten unabhänig, auf einem Herauslösen bildnerisch reiner Beziehungen*" [Klee 1964];

Abstract Concepts

The above requirement (a) can be formalized with the help of the cognitive framework of chapter 1. Thus, the question remains, what special kind of entities are depicted by abstract pictures, i.e., what type are the entities the visual algorithms maps to; without anticipation, these will be called abstract concepts. Hence, an *abstract picture or diagram* depicts abstract concepts.

abstract pic. or diagram

The following introduction will avoid the discussion of the ontological status of abstract entities, because these concepts are defined via properties of their perceptional mapping which lack a formal foundation. The question about the ontological status of these entities leads directly to the controversial philosophical discussions of the existence of universals [sep:universals-medieval], of the concrete-abstract dichotomy [sep:abstract-objects], and of a formalized notation of conceptual generalization which was already applied in def. 2.10[26]. Following from requirement (a) together with def. 7, abstract concepts cannot be sub-concepts; hence, the act of composing abstract concepts from other concepts gets into focus. Without having introduced metaphors and their relation to sub-concepts [Lakoff & Johnson 1980], which would be a first step towards the analysis of abstract concepts' origin, at least a first working definition can be extracted:

[26] for example, [Herre et al. 2006] based GFO's definition of «abstract» on [Hartmann 1940] and [Ingarden 1964];

(Preliminary) Definition 6.1

abstract concept

> *Abstract concepts* are those concepts of a conceptual space that are neither sub-concepts directly nor derivable by a spatial composition of patterns of sub-concepts.

In the following, this simple preliminary definition will have enough definitory power to classify the given prototypes when modelling the diagram domain, even if it seems unsatisfactory from a theoretical point of view.

Example 10 : Extracting an Attribute (continued)

As abstractness is an important property of a large class of diagrams (see discussion of prototypes), it has to be included as an attribute in the FCA modelling. In order to decide whether an object exhibits «abstractness», the previous formal foundation via a definition that is based on the cognitive model and conceptual spaces suggests an appropriate decision procedure: a prototype has the property of abstractness iff it depicts abstract concepts, i.e., the cognitive algorithm maps the perception of the picture to these concepts.

the property of abstractness

decision procedure

Before applying the property of abstractness to conceptual modelling, the subsequent paragraphs will introduce other important aspects regarding pictures and existing conceptualizations.

From Gestalt-Semiotics to Semantics

Summarizing the given approaches to pictorial semantics, semantics is no singular phenomenon but is heavily based on the underlying cognitive model and the other layers of the meta-model. There are two possible ways of approaching the mapping of pictures to their intended meaning: first, one can analyze this mapping, i.e., the labelling relation (cf. p29), or, second, investigate the process itself which does this assignment. Regarding part I of this work, the first category of properties is about semantics in the sense of the linguistic framework whereas the second emphasizes cognitive psychology. Whether these linguistic categories can be actually applied to pictures will be analyzed afterwards.

Avoiding Resemblance by Literal Preservation

Putting aside the discussion about similarity and resemblance, the quality of the labelling relation includes the structural isomorphism between the picture and the depicted. Eric Hammer describes this as the contrast of *literal vs. non-literal preservation* [Hammer 1995]. In the case of literal preservation, the relation between the diagram's semiotic entities is *literally* the same as between the depicted objects.[27] Hence, literal preservation plays a central role in presentational pictures, whereas diagrams restrict the focus to certain features of the depicted object. These are only encoded by certain graphical propositions. Hammer's classical examples are Venn diagrams (cf. p67 and the discussion of Euler diagrams at p99) which use propositions about circles to depict propositions of the underlying sets whereby not all what is true to circles is true to sets; another example are circuit diagrams which are only topologically connected to the depicted domain but do not show actual wirings – they

literal vs. non-literal preservation

[27] Umberto Eco advocates the recognition of this literal resemblance as being part of convention because the distinction between properties of the picture itself and the depicted properties is not clear; therefore, literal preservation is no usable touchstone for the classification of diagrams (this was introduced and opposed in [Hammer 1995, ch. 1]);

are an abstract-graphical representation of an electronic component, e.g., the diagram at p69 represents the general category of a resonant circuit.

Attentative and Pre-attentative Vision

Bernd Weidenmann combines the pragmatic area of educational pictures with research in cognitive psychology. Whilst introducing a psychological model of understanding pictures [Weidenmann 1988], picture cognition can be – in analogy to the difference between pre-concepts and concepts – either *pre-attentative* or *attentative*. Pre-attentative pictures allow for the fast[28] (re-)cognition of the depicted as a whole but depend on concise graphical codes, conventions, and enough previous knowledge. Attentative pictures must be read in detail but allow for the introduction of new notions and information as well as synthesizing knowledge out of a picture which was not explicitly encoded by the author. Thus attentative pictures favour *synthetic reading* as opposed to a strictly *analytic reading* [Schnotz 1993] which only extracts the encoded data.

pre-attentative vs. attentative

[28] 'fast': in about $\frac{1}{10}s$ [Weidenmann 1988, p28];

synthetic vs. analytic reading

To summarize, the properties pre-attentative and attentative allow for a psychologically based notion of a picture's readability.

Example 10 : Extracting an Attribute (continued)

> The difference between attentative and pre-attentative reading as well as the property of illiterative preservation allows to derive a decision procedure for the subsequent modelling. In contrast to the previous definition of «abstractness», these approaches lack the formal foundation of the underlying cognitive model. Nevertheless, an elaborate investigation of the differences and interrelations between these cognitive notions would reveal important aspects of the underlying concept of «abstractness».

Semantics Precedes Syntax

As already introduced in sect. 3.2, the differentiation between syntax and semantics is only artificial but plays a certain role when introducing formal languages (def. 2.8) based on the principle of compositionality (def. 2.9). As the principle of compositionality is central to a natural language's grammar, syntax[29] is normally thought to precede semantics; a picture's "closeness" to perception blurs this distinction even more because semantic features are directly depicted by pattern composition and, vice versa, the structural features of a pattern are mapped directly to sub-concepts. Consequently, this subordinate role of syntax is often described as "*semantics precedes syntax*", opposed to the leitmotif of classical formal language; nevertheless, the requirement of a formal syntax which will enter the discussion via formal dia-

syntax and semantics

[29] following [Plümacher 1998], pictorial syntax is the visual structure that matches the structure of the depicted object;

grammatic languages in sect. 7.3. For pictures in general, this linguistic distinction will be avoided[30] and the pattern-based compositional aspect will be captured by the Gestalt-semiotics layer.

[30] [Gerhardens 1998] & [Plümacher 1998] even neglect the existence of something like (classical) syntax for pictures at all;

6.2.3 Pragmatics

Most pedagogic and educational approaches center around the role of pictures and diagrams in learning and understanding. As learning cannot be separated from cognition, most pragmatic categorizations focus on cognition. Nevertheless, one can categorize pictures due to the intention of the author into *educational pictures* that are used in classical education, *scientific pictures* as a part of the research activity, and *knowledge pictures* that are merely ways of storing and expressing knowledge [Schnotz 1993].

educational vs. scientific vs. knowledge pictures

Bernd Weidenmann introduces a general categorization of pictorial presentations into the classes of *informatory pictures*, *artistic pictures*, and *entertaining pictures* [Weidenmann 1993a] whereas Schnotz's trichotomy above can be subsumed under informatory pictures. Informatory pictures are constructed to make assertions in instructional situations, e.g., in the context of education. Therefore, these pictures should be unambiguous and cover the domain completely. On the contrary, artistic pictures fall under the hypothesis of the uninterpretability of aesthetics. Entertaining pictures only aim at arousing emotions. Nevertheless, both can be utilized in an informatory context.

informatory vs. artistic vs. entertaining pics

Tufte's Evidence Presentations

Edward Tufte regards pictures and diagrams as presentations of *evidence* [Tufte 2006b]. The idea of evidence explicates the transformation (see Küker p62) of intense seeing that generates empirical information into the "*showing*" of explanations and evidence. These *evidence presentations* form a category that is solely based on pragmatics analogous to the informatory pictures of Weidenmann. Tufte introduces another important category of diagrams: *mapped pictures*, which combine representational images with scales, other diagrams, overlays, and numbers (see the AFM plot prototype (p66) or the weather chart (p68)). Therefore, they combine "*the direct visual evidence of pictures with the power of diagrams*" [Tufte 2006b, p40]. The underlying pictures are representational, local, specific, and unique objects compared to the contextualizing and abstractness of diagrams.

evidence presentations

"showing"

mapped pictures

After introducing sparklines – word-sized graphics that are embedded into the flow of the text (heterogeneity type (d)) and depict a simple timeseries [Tufte 2006b, p46ff] – he focuses on another important category of diagrams which link nodes by arrows which represent a causal relation. In the following, this type of diagrams will be named *arrow-and-node diagrams*. They subsume the causal loop diagrams, Feynman diagrams and semantic nets and hence are broadening the scope of causality to general interrelations.

arrow-and-node diagrams

6.2.4 Bertin's "Semiologie Graphique"

The most profound analysis of diagrammatic representation was published by Jacques Bertin in [Bertin 1973][31]. Although his approach lacked a formal definition of his basic graphical units – the *graphic representations* or "graphiques", he was able to deduce an axiomatic system that described the different classes of diagrams by simple icons.

[31] all references including a page number are to the German edition [Bertin 1982];

His most basic entities are *graphic representations* which are visually perceivable, meaning-bearing forms that can be perceived instantaneously [Bertin 1982, p150]. These forms draw a distinction on the plane of the diagram which is homogeneous and continuous, and which are perceived as *spots* that are categorized by contextual knowledge. These graphic representations map data variables, e.g., of empirical analysis, to visual variables. *Visual variables* are the two dimensions of the plane and the additional ways to differentiate these 'spots' by size, colour, form, brightness, and pattern [Bertin 1982, p50]. Visual variables, i.e., a triangular spot at a certain region in the plane, are perceived as something that represents a real-world variable which can be further categorized due to its quality: it can be selective (≠), associative (≡), ordered (o), or quantitative (Q). Quantitative variables bear values of a realm of real numbers, thus inherently include a measure. The other categories describe the relations between the different instantiations of the same spot. Considering two (perception-)equal spots on two different regions on the plane, these represent either the same entity (≡) or different entities of the same type (≠) which can both be further ordered (o)[32].

graphic representations

spots

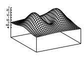
visual variables

depicted vars. $\neq, \equiv, o, Q$

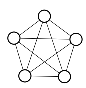
[32] the difference between o and Q resembles the difference between a (finite) subset of the natural numbers and the real continuum;

Next, Bertin distinguishes three basic types of graphic representations: (a) *diagrams* that represent relations between *all* elements of one component and *all* of the others; 3 D function plots or the AFM graph are examples of diagrams depicting three qualitative components; (b) *networks* represent all relations between all elements of *the same* components, e.g., all relations between all nodes of graph as in the graph representation prototype above; and finally, (c) *maps* include a geographically measurable component which is represented by the geographic measure of the plane and a network which is initially ordered, e.g., a weather chart, which depicts a geographical region and additionally represents the selective component of air pressure by symbols.

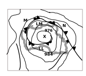
diagrams (Bertin)



networks



maps



Summarizing these ideas, Bertin categorizes graphical representations by (i) the number of used visual variables, (ii) their *imposition*, i.e., their utilization of the plane's 2 D space, (iii) the type of the depicted variables, i.e., their *implantation*, which is the way the 'spots' are perceived as either points, lines, or planes and (iv) their *length*, i.e., a subjective measure of a component's cardinality. The imposition is a pair of one of the three different imposition bases (diagrams, networks, maps) and the type of the occupation of the plane as either linear $\longrightarrow$, circular ↻, areal, areal with underlying order, or using orthogonal projection. Visual variables like colours, brightness, etc., are depicted as dimension "out" of the plane ↗. This leads to a diagrammatic symbol system which allows to represent each class of diagrammatic representations with a specific icon. This will be shown in the following example.

imposition

implantation

length

Example 11 : Bertin's Categorization

Fig. 6.4 depicts the results of a comparative processor benchmark taken from [Riepe 2000]. The starplot in the left of the figure presents the results of twelve test runs (o as they are distinct, and given in a circular ↻ order) as "rays". These test runs result in a quantitative (Q) time measurement which is represented by the length of the ray in a linear way $\longrightarrow$, additionally the average result is given by the dotted circle (Q). Since one compares n different results side by side ($\times n \longrightarrow$), the icon on the right of fig. 6.4 graphically represents this class of diagrams.

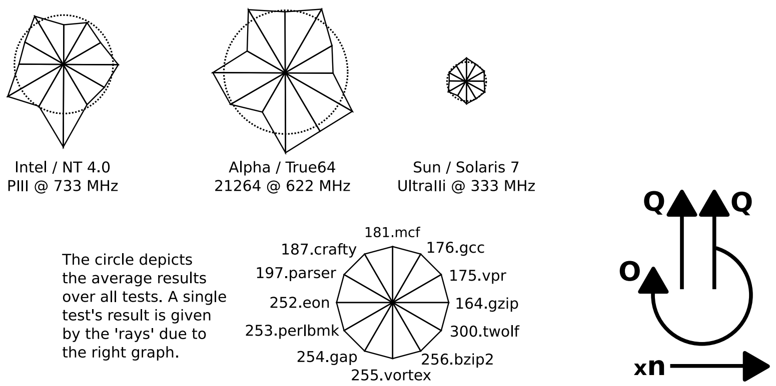


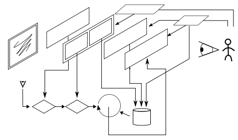
Fig. 6.4: Categorization of the Result of a Processor Benchmark (SPECint2000) depicted as Series of Starplots taken from [Riepe 2000] with the Symbolic Representation due to Bertin's System

To conclude, Bertin's system allows to categorize a special kind of diagrams, those which can be described by data-mappings in a very detailed fashion on the Gestalt-semiotic layer. Further, his classification supports a large variety of diagrams that are used in information design and compared in [Bertin 1973]. This special kind of graphic representations will be denoted by *data-mapping diagrams* which also include Tufte's evidence presentations.

data-mapping diagrams

6.2.5 Summary of the Different Approaches

To conclude, there is a large literature about pictures and diagrams especially regarding the research of Bildwissenschaft. Recapitulatory, most conceptualizations lack a formal foundation which would allow to compare and interrelate between them as well as to derive a decision procedure for prototypes. The example of «abstractness» revealed the necessary steps which would have to be applied to all the given properties in order to proceed with the modelling.



(fig. 5.2)

The meta-model allows to categorize the different approaches and to embed at least their background into the formalized notion of cognition. Most theories only describe one aspect of pictorial presentation, like database models which mainly describe physical properties (p58); others describe the relation between layers, e.g., the (denotational) semantic approaches which try to describe the relation between both the physical and the symbolic layer to meaning.

resemblance & abstraction

Nevertheless, all the approaches have a great deal in common. The peculiarity of pictures depends on both a gradual inclusion of resemblance into semantics and a certain level of abstraction, either as abstracting data from a causal relation of the real word (cf. diagrams depicting abstract data), as depicting abstract concepts (by text labels), or as metaphorically utilizing a diagram's space to represent abstract relationships. Further, the extraction of both Gestalt and symbolic basic units aims for the description of a certain pictorial language with the help of a grammar, thus allowing to introduce *iconic languages* as an extension of classical, sentential (natural) languages.

iconic languages

The next step will try to extract a formalized conceptual basis for the domain of diagrams that includes this basic insights.

6.3 A Rudimentary Fca Approach – Clustering the Prototypical Domain

The previously introduced prototypes and theories about diagrams constitute a large base for Fca-based conceptual modelling as introduced in appendix F. This procedure depends on a pre-formalization of the domain into prototypes and properties; the latter will be extracted from already existing (pre-)conceptualizations, e.g., the property of «abstractness» as formalized in this chapter's main example.

The modelling recipe tries to generate step-by-step an Fca-context that expresses the incidence between objects and attributes where Fca-objects correspond to prototypes and an Fca-attribute is based on a formalized notion of a corresponding property. The creation of the lattice is described as a card game that explores the conceptual search space whereas each snapshot of the game is equivalent to an FCA-context (viz. fig. 6.5).

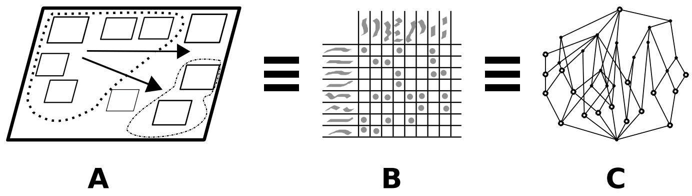


Fig. 6.5:
From a Snapshot of the Modelling Game (A) to the corresponding Hasse Diagram (C) via the Context Table (B) (cf. fig. F.1)

The underlying modelling recipe is introduced in detail in appendix F; the following sections will only present two resulting FCA lattices which will continue the exemplary discussion of «abstractness». These results will prototypically illustrate the problems of modelling the diagram domain with this methodology; consequently, two other ways to approach the diagram domain will be presented that include the FCA results.

6.3.1 Two Practical Examples of FCA Modelling

As already emphasized, the following two examples will only present intermediary snapshots of a modelling game, not the sequence of moves that lead to this outcome. Both examples will show how objects and attributes are extracted from prototypes and pre-conceptualizations which were already presented in the previous sections. Further, the example property «abstractness» will be tracked to its representation in Hasse diagrams.

Formalizing Main Results of the Previous Discussion

In the first example game, the set of formal attributes is given by the following properties: the difference between "quantitative data" and "qualitative data" expresses the quality of the depicted data which can either be quantitative like Bertin's category Q, and thus has a measure, or allows for qualitative comparisons only ($\neq, \equiv, o,$); "abstract categories", "process abstraction", and "topological measure" derive from Schnotz's three basic categories of pictures; "synthetic reading" allows to extract information from the picture which was not included by the author directly but can be derived by the reader; "non-perceptive properties" describes the presence of entities depicting abstract concepts. The relation between a diagram that exhibits non-perceptive properties and its subsumption under Schnotz's class of diagrams that depict abstract categories or processes is left open. A possible snapshot of the modelling game is presented in tabular form in fig. 6.6.

property of abstract categories (ex. 10)

	quantitative data	process abstraction	topological measure	abstract categories	synthetic reading	non-perceptive properties	qualitative data
plot of fct.	×		×				
circle D	×						
Feynman D		×	×	×		×	×
3D eng. draw.	×		×				
expl. D			×				×
ortho. proj.	×		×				
cross sect	×		×				
road map	×		×				
weather chart	×		×				
route sketch		×	×	×			×
subway map			×			×	×
circuit D			×			×	×
free-body D	×		×	×	×		
Venn D							×
tree D						×	×
semantic net				×	×		×
causal loop D		×		×	×	×	×
assembly D		×					×

Fig. 6.6:
Example FCA Lattice 1

Discussion of the First Game

The representation of a context as a Hasse diagram in fig. 6.7[33] allows to draw first conclusions: (a) Schnotz's trisection does not partition the given set of prototypes (viz route sketch); (b) the differentiation of diagrams regarding the quality of the depicted data reveals to be a basic distinction; (c) the 'supernode' including most engineering diagrams needs further attention; (d) causal loop, Feynman, and free-body diagrams share a certain level of detail.

[33] this diagram was generated from the lattice by CONEXP `http://conexp.sourceforge.net`;

Example 10 : The Attribute of Abstractness (continued)

Fig. 6.7 reveals «abstractness» to be an important attribute in the FCA lattice because the attributes of being a process or category abstraction are located near ⊤. Further, the attribute of non-perceptive properties limits an important sublattice of objects whose prototypes were previously classified as abstract diagrams. At the step from the previous a priori understanding of abstractness

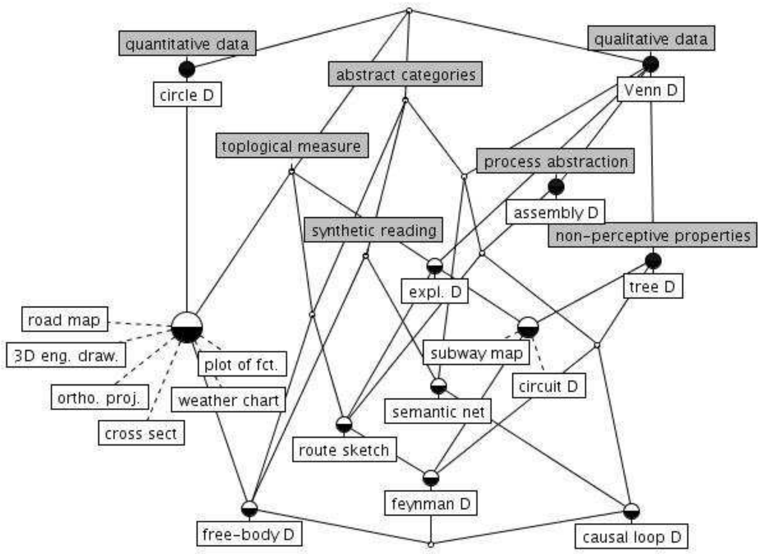


Fig. 6.7: Hasse Diagram of FCA Lattice 1

to the node in the Hasse diagram, the requirement of a formal foundation of these properties (viz def. 6.1) and a decision procedure that is based on this foundation becomes justified. In order to construct the incidence relation, the modelling engineer has to decide whether a certain attribute inheres in an object, i.e., whether a prototype has a certain property[34]. This decision has to be based upon a deterministic decision procedure.

[34] FCA would allow to represent attributes that are not binary but can take certain values, these can be reduced to binary predicates due to [Ganter & Wille 1996]; here, only binary predicates will be considered;

First Steps towards a Basic Categorization

The second game tries to deal with the shortcomings of the previous results. Hence, the supernode will be dissected with the help of goal properties, i.e., binary predicates that represent a classification of objects (see p178); goal properties help to formalize intuitions about categories by introducing these as a binary property of «being-a-member-of-this-category» [35].

goal properties

[35] this modelling trick depends on Frege's dualism of denotation and sense that underlies FCA and allows to describe a real world object by both its extension as a class and the properties that describe the underlying sense;

These three goal-predicates are based on the classification that underlay the introduction of prototypes (p65ff) that was given by the dissection into «technical diagrams», «maps», and «diagrams» (in the following, abbreviated by 'D') whose membership is expressed in the appropriate column of fig. 6.8.

Other properties that are applied in the modelling game represent a categorization of the depicted object ("space vs. time vs. data"), the usage of "abstract" concepts and the usage of text to represent these entities in a heterogeneous way ("abstr. txt"), the need for a "legend", and the usage of visual "metaphors" which will be later intro-

abstractness (ex. 10)

	close to perception	3D	space	time	data	abstract	abstr. txt	legend	metaphoric	goal: D	goal: techD	goal: map
painting	×	×	×									
photo	×		×		×							
radiograph			×		×			×				
sonograph			×		×			×				
AFM		×			×			×				
plot of fct.		×						×				
circle D												
Feynman D				×		×	×			×		
3D eng. draw.	×	×	×					×			×	
expl. D	×	×	×								×	
ortho. proj.		×	×								×	
cross section			×								×	
road map			×			×	×					×
weather chart					×	×	×					×
route sketch			×	×		×	×					×
subway map			×			×	×					×
circuit D												
free-body D				×		×	×			×		
Venn D						×			×	×		
tree D						×	×		×	×		
semantic net						×	×		×	×		
causal loop D				×		×	×		×	×		

Fig. 6.8:
Example FCA Lattice 2

duced as free rides; the property "3 D" represents the dimensionality of the diagram or picture[36].

As depicted in fig. 6.10, the class of map-like representations is grasped by a hybrid concept that depicts a region of space with an additional layer of abstract data. The concept «diagram (D)» does not exhibit this connection to the real space, but shows a restricted sub-concept: the diagrams that additionally depend on a metaphoric "misuse" of the space represented by the diagram's plane. Further, time is depicted with the help of abstract graphical features, e.g., arrows. The prototypes photo and painting can be described as space representations close to perception, i.e., depicting spatial properties in a "realistic" manner. Unfortunately, circuit and circle diagrams are not classified by the given FCA-context; similarly, a function plot's vicinity to 3 D engineering drawings surprises at a first glance.

[36] whereas a more detailed description of the representation of 3 D, e.g., via perspective, would be more appropriate; cf. sect. 1.6;

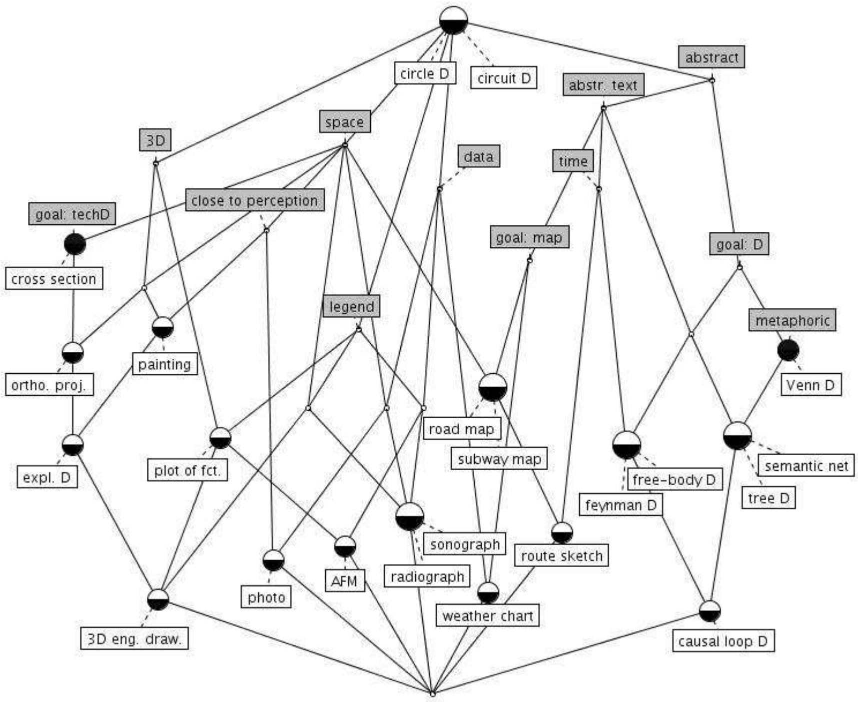


Fig. 6.9:
Hasse Diagram of FCA Lattice 2

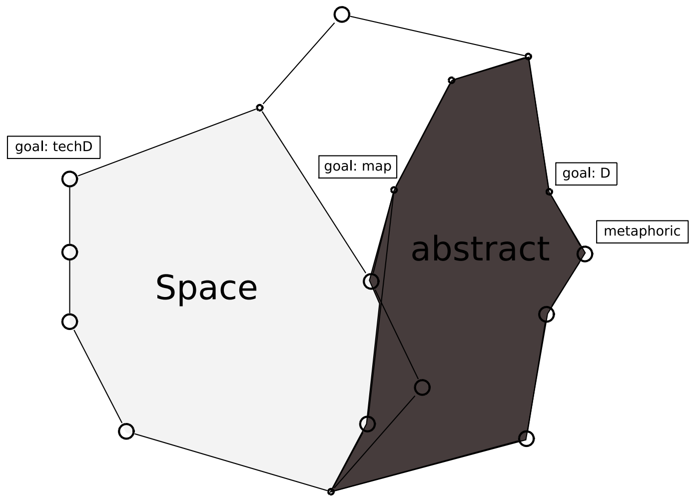


Fig. 6.10:
Important Aspects of the previous Hasse Diagram

Example 10 : The Attribute of Abstractness (continued)

The previous discussion highlighted the important role of «abstractness» in the domain of diagrams. Hence, diagrams rely on abstractness whereas a further distinction towards maps seems inevitable.

As already mentioned, the previous two snapshots are to be seen as the prototypes for a series of modelling games that were all unable to extract a conceptualization which included both the majority of prototypes as well as the basic understanding of diagrams that was made explicit when introducing prototypes and properties. The most common shortcomings were already shown in the above examples: the resulting model was either unable to categorize important prototypes (cf. circle diagrams in Fca lattice 2) or revealed – from the point of view of common understanding – strange subsumptions (engineering drawing under plot of a function). The next section will discuss why these flaws were inescapable.

6.4 Modelling highly Multi-dimensional Domains with Fca

A correct proof of why the modelling game cannot result in an all-embracing model of the diagram domain depends on a formalized notion of the game and, especially, a termination condition, i.e., the reformulation of the subjective measure of a "best" matching between a specification and its underlying domain as the goal of the game. Anyhow, a brief meta-analysis will point out certain factors that sabotaged the simple modelling recipe from the start. Nevertheless, the results of the above examples will become become become become become become become become become the starting point of two other modelling approaches.

lack of termination condition

6.4.1 Why the Modelling Recipe had to Fail

A view onto the meta-model of sect. 5.3 could have avoided the naïvety above: as the diagram and picture domains are highly multi-dimensional and the given pre-categorizations of sect. 6.2 were not able to rudimentarily propose satisfactory sub-models of even minor aspects of this domain, an attempt to include all these different, partially conflicting, and interconnected aspects in one single model is impossible as a detailed analysis would depend on a prior formalization of the pre-categorizations that originally lacked any mathematical rigour.

counter-examples

Thus, the existence of counter-examples or prototypes that cannot be subsumed by the given conceptualization (viz above example lattice 2) seems natural, since the gargantuan size of the domain eases to find these. Further, as the discussion of the theories that underlie the properties already highlighted, these properties are interconnected and can often not be separated entirely from each other, e.g., «abstractness» can be related to either «attentive reading», Schnotz's classes of categorial and process abstractions, or «illiteral preservation». Hence, each context-lattice, which is built in each step of the modelling game, includes inherent relations between Fca-attributes which would had to be made explicit in advance which, again, depends on a rigorous foundation of the underlying theories.

inherent relations between attributes

6.4.2 Extending the Results beyond Fca

There are several ways to face the above discrepancies which either extend the Fca approach or transfer the previous, intermediary results into another modelling paradigm.

The advantage of the modelling game is its simplicity and the possibility to include a formal fundament via Fca-lattices. Hence, an fundamental enhancement of the proposed modelling recipe which includes research results from the modelling of highly multi-dimensional domains with Fca could be a possible starting point. Another shortcoming is this lack of formalization and formal foundation for the properties which the above example games used in an intuitive, common sense manner.

enhance Fca-game approach

lacking formalization of properties

As this procedure would include a rigorous reformulation of the game in terms of game-theory, the following proposal will focus on other modelling paradigms.

First, the domain will be restricted to "abstract logical diagrams", e.g., conceptual graphs, Uml diagrams, and semantic nets, which play an important role as conceptual modelling languages. Second, the usage of axiomatics (cf. def. 2.12) overcomes the lacking formalization of underlying properties as these are included in the axioms just as well. At least, this will result in in a formal basis for the subsequent part's analyses of conceptual graphs.

axiomatics

Nevertheless, a more general approach towards this domain is inescapable. Sect. 8 will combine the results of the first part with the previous findings, and attempt to gain a formalization with the help of category theory. As this approach would include a formal model for the cognitive model as well and depend on sophisticated mathematical modelling, only a draft will be given.

category theory

7 Introducing Logical Diagrams: A Simple Axiomatic Approach

Summarizing the previous discussion of the shortcomings of the proposed FCA-modelling method as well as recapitulating the presented pre-conceptualizations above, the following axiomatization will try to extract the essence of being a diagram. The main inspiration recaptures the idea of a picture as a "transformation machine" enrooted in Bertin's approach (p95) as well as Küker (p62), Tufte (p84), and Deleuze (p79); and further, the importance of «abstract» as applied in the utilization of linear text in diagrams, i.e., words, that represent abstract concepts[1] and the opposition towards presentational depiction.

[1] heterogeneity (p73) of type (f);

7.1 Axiomatic Introduction

Analogously to the well-known bisection of graphic programs into raster and vector based, the picture domain can be categorized with the help of the basic Gestalt-semiotic building blocks. Contrary to the first distinction which is merely a question of the underlying file-format and a program's way of interfacing the picture due to the possibility to convert[2] from raster into vector formats and vice versa, the above distinction allows to differentiate the basic *meaning bearing* building blocks which will be called *graphical elements*.

[2] there is no possibility to translate both formats 1:1 at each scale, as raster images correspond to a fix resolution which cannot be changed without loosing pixel information;

graphical elements

Regarding the carrier of the picture, the most fine-grained visual meaning-bearing units are pixels of a certain resolution (think of the colour dots of the CMYK printing), whereas these can be agglomerated into points and lines which are the basis of Gestalt extraction. The choice of the appropriate granularity resides in a picture's pragmatic dimension and therefore its embedding in a communicative act. Further, graphical objects have to be distinguished from Gestalt perception's basic patterns, as will be shown in the next example.

7.1.1 Graphical Elements

Graphical elements form the basic units of pictorial presentation. Thus, their entirety constitutes the *signature* of a picture. There are different approaches to introduce

signature

these basic entities: Bertin's spots (sect. 6.2.4), Klee's "genesis of form" which reduces graphical entities to the movement of a point[3] [Klee 1964], or linguistic approaches which are discussed in [Sachs-Hombach 1998c].

Regarding this thesis' cognitive approach, spots that range from colour points to Gestalt patterns are the basic graphical entities ("graphiques") of perception. For example, these graphiques are either lines, curves[4], arrows, or simply dots and areas. In heterogeneous formalisms, characters as either symbols, icons, or – regarding granularity – words complete the repertoire of the basic elements of a visual language. The central role is played by the Gestalt-semantics layer, i.e., the Gestalt pattern matching step that results in the input to the cognitive search algorithm.

[3] the movement of a point results in a line; an area can be generated as a fabric of lines; areas form surfaces of solids; this is analogous to the step from 0 D to 3 D in Euclidian geometry;

[4] a mathematical approach would demand additional features like continuity to avoid special cases like the Koch snowflake to obtain "cognitive adequate" basic entities, i.e., graphical basic units that are pre-attentative and finitely representable;

Graphical basic elements have to be Gestalt patterns that are perceived as a whole. Additionally these basic units have to be differentiable (cf. def. 2.7) to avoid ambiguity.

Example 12 : Ambiguity of Graphical Entities

Good diagrammatic design has to avoid the ambiguous usage of graphical elements as exemplified in fig. 7.1.

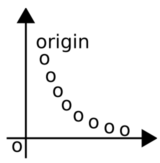
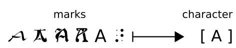


Fig. 7.1: Ambiguous Function Plot

The spot "o" is used to depict at least three different entities: (a) a point that represents a tuple of data, (b) the number zero to represent the measure of the point where the two axes meet, and (c) the first letter of the word "origin". Recognizing these different possible usages of the spot "o" depends on knowing the conventions of function plots to depict points by dots (a) and to give units of measurements (b); identifying the top "o" as a letter (c) depends on a process of elimination as the word "rigin" is no word of the English language[5] contrary to "origin"

[5] additional background assumptions: the underlying language is English and this plot is not embedded in a biochemical context about the synthetic peptine Rigin (glycyl-L-glutaminyl-L-prolyl-L-arginine);

This is analogous to the role of background knowledge in sect. 1.5 and the classification of marks to characters (fig. 2.7), though. With graphical elements as first class symbols, there are more possibilities to match a graphical element.



(excerpt of fig. 2.7)

7.1.2 Abstract Diagrams

Based on graphical elements, diagrams are specializations of pictures which were formally introduced by axiom 1 (closeness and semiotics) and postulate 8 (plane, lasting objects).

Definition 7.1

diagram

A *diagram* is a picture whose graphical elements are abstract forms or vectors, i.e., points, lines, patterns, and arrows with the additional help of word-labels (heterogeneity).

Hence, this definition of diagrams only restricts the domain of pictures by the underlying symbol system of the Gestalt-semiotic layer. These diagrams include most of the diagram prototypes[6] of sect. 6.1. The next definition will further narrow down this definition to the layer of cognitive semantics and attach a notion of abstractness.

[6] simply, all prototypes with "diagram" in their name;

Definition 7.2

abstract diagram

A diagram depicting mainly non-perceptional, abstract concepts (def. 6.1) and their interrelations with the help of perceptual metaphors is an *abstract diagram.*

Example 10 : Abstractness: from FCA to Axiomatics (continued)

The ingenuity of this definition resides in the usage of perceptual metaphors, i.e., the symbolic usage of graphical elements to express relations between abstract concepts[7]. These abstract diagrams are objects composed from basic building blocks (arrows, word-symbols, etc.), and thus reminds of the compositional approach (def. 2.9) which is central to formal languages (def. 2.8).

[7] cf. the discussion of «abstract» (p80ff), especially Klee's definition of abstract art;

An explicit statement of the labelling relation of basic building blocks together with a compositional grammar[8] leads to a formal language based on a special type of diagrams which are attributed as abstract logical.

[8] here, «grammar» is used, to avoid the split into syntax and semantics which would complicate the "semantics precedes syntax" leitmotif (p83);

Definition 7.3

diagrammatic formal language

A *diagrammatic formal language* is a formal language in which the language's signature is a set of graphical elements and the entities composed by the underlying compositional grammar[8] are abstract diagrams.

Definition 7.4

abstract logical diagrams

Abstract logical diagrams are abstract diagrams that are based on a diagrammatic formal language and thus have a fixed formal semantics.

The formal language framework of abstract logical diagrams allows to introduce a deduction system which can be based on visual inference rules only[9]. A classical example are existential graphs and their extension to conceptual graphs which will be introduced in the next part (visual deduction is presented in appendix E).

visual deduction

[9] an introduction to general reasoning with diagrams is given in [Wang et al. 1995];

Definition 7.5

Abstract logical diagrams in a strong sense are abstract diagrams that are based on a diagrammatic formal language whereas the $\vdash$-rules are given in a diagrammatic way only.

abstract logical diagrams (strong sense)

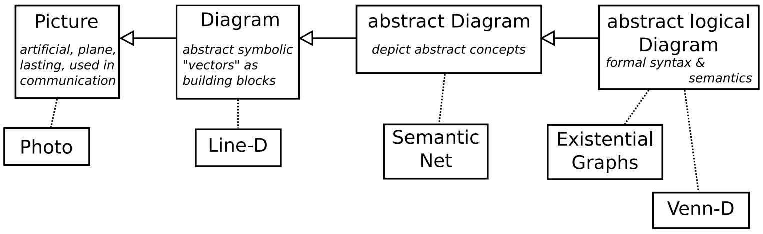


Fig. 7.2: Diagram of the Relation between the previous Definitions

Fig. 7.2 summarizes the main results of the axiomatic modelling: def. 7.1 introduced a semiotic distinction to derive diagrams from the general concept of «pictures», the next definitions transferred the idea of formal languages to diagrammatics (def. 7.3) through the backdoor of abstract logical diagrams (def. 7.4). The basic axioms were the introduction of pictures (post. 8) and the paraphrase of basic graphical elements.

The central object of the presented models are abstract diagrams and their subspecies with a formal language background. Abstract diagrams can be compared to Schnotz's logical pictures ("logische Bilder" [Schnotz 1993]) and the diagrams of Hammer [Hammer 1995] whose underlying intuition resembles the previous discussion but lacks a rigorous formalization.

7.2 Sentential vs. Diagrammatic Formal Languages

In abstract logical diagrams, graphical signs are applied with a fixed, compositional semantics, and, hence, resemble classical symbols of linear text whereas the diagrammatic arrangement allows for graphical free rides via visual metaphors.[10] Consequently, the main difference between sentential, linear text based formal languages and diagrammatic ones resides in the additional usage of graphical representation. As will be exemplified when translating conceptual graphs to formulae of Fol in sect. 12.1, both formalisms are able to express the same meaning regarding the underlying formal semantic foundation. Nevertheless, there are certain aspects of diagrams that exceed linear language and are advantageous for practical (conceptual) modelling.

[10] visual free rides were already basic to Arnheim's ideas: the topological features of diagrams allow to need less 'space' for the storage [Arnheim 1969];

7.2.1 Free Rides

Diagram's central superiority is the usage of pre-attentative, graphical features in *free rides*, a term coined by Atsushi Shimojima [Barwise & Shimojima 1995] [Shimojima 1996]. Free rides are conntected to the ideas of illiterative representation (p82) and synthetic reading (p83).

free rides

Example 13 : Free Rides

The following Euler diagram was originally drawn to depict the inclusion of set B in A ($B \subseteq A$) and the fact that the sets A and C are disjoint ($A \cap B = \emptyset$).

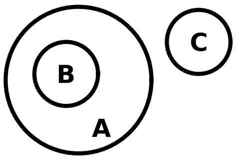


Fig. 7.3:
Free Rides in Euler Diagrams

These relations are expressed with the help of visual constraints, i.e., a distinctive usage of graphical features regarding and underlying visual language, in this case: Euler diagrams. But Euler diagrams allow to directly derive – in a pre-attemptive way – an additional fact: B and C are disjoint ($B \cap C = \emptyset$).

This is not derived with the help of any explicitly stated graphical deduction rules of the formal language of Euler diagrams but depends on the visual metaphor that underlies the introduction of this graphical formalism: the use of nested circles to depict set inclusion as graphical containedness.

Hence, free rides can be traced back to basic geometric sub-concepts which can be read in a pre-attentive way. Every diagrammatic representation allows for free rides but not all diagrammatic languages[11] utilize this "free" and fast way to express relations.

Regarding the visual metaphor of example 13, not all what is true for circles is true for sets[12], but to what amount the features of the underlying graphical entities influence the reader's inferences on the depicted objects cannot be made clear in advance as it depends on visual literacy→ and an a priori understanding of what could be depicted by free rides.

Hence, on the one hand, free rides allow for a compact, pre-attentatively readable representation; on the other hand, they depend on the reader's subjective decision which basic graphical features are the base for inferring knowledge.

The famous edition of Euclid's "Elements" by Oliver Byrne in [Euclid 1847] utilizes free rides to visualize basic geometric proofs without introducing an explicit diagrammatic deduction system. Each step of a proof is based on the illiterative

[11] there are rare examples of free rides in linear language, like the sentence "Pythagoras is twice as large as Jones" [Hammer 1995, p6];

[12] this was already discussed when differentiating Hammer's literal and illiteral preservation (p82);

depiction of the underlying geometric properties by visual properties of the corresponding diagram. These diagrams are intuitively readable due to this "closeness". [13]

[13] or in the case of an engineering sketch "*... a thousand constraints*"[Stahovich 2002];

7.2.2 The Fallacy of 1000 Words

The most common misunderstanding is couched in terms by the famous quote "*a diagram is worth ten thousand words*", to which Jill Larkin and Herbert Simon add an important "*sometimes*" [Larkin & Simon 1987]; and Alan Blackwell even restricts this expression to "*... is worth 84.1 words*" [Blackwell 1997][13]. A diagram *can* be worth more than a linear representation, especially when communicating scientific results, but it does not per se substitute 1000 words. The central aspect is the diagram's visual *quality* [Weidenmann 1993b] [Hammer 1995]. Hence, the supremacy of diagrammatic presentation depends on the author and recipient's abilities to produce and understand "good"[14] diagrams. "Good" diagrams extend "good" linear text with the help of graphical free rides, which allow for an easy readable, compact[15] presentation of information.

[14] as already stated, quality measures are important to pictorial presentations but out of this thesis' focus;

quality

[15] here, a general measure of the compactness of information could be based on the notion of information-equivalence, cf. information→;

7.2.3 The Role of Diagrams in (Scientific) Modelling

Hence, regarding the simple translation of abstract logical diagrams into other knowledge representation formalisms, which is possible due to their common underlying formal semantics, the discussion of a diagram's expressive quality has to be restricted to the pragmatic background of a concrete communication situation, as well as the readability and the visual literacy→ of the reader[16]. The quality of a picture or text is a *subjective* measure (cf. [Pirsig 1974]) based on personal experience and learning. For example, reading and extracting information from a Hasse diagram, as used to represent FCA lattices above, needs a first reading tutorial and subsequent practical experience.

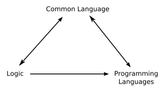
[16] cf. for visual literacy in the context of programming see [Petre 1995]; for general geometric literacy see [Koedinger & Anderson 1990];

As stated by hypothesis 7 (basic modelling), sketches bridge the gap between the perception of the real domain and a formalized notion. They are *the* basic tool in practical science, as Daniele Bailer-Jones argues for "*sketches as mental reification of theoretical scientific treatment*" [Bailer-Jones 2002]. And with the help of the representation of abstract entities, these sketches play a central role in highly abstract research domains as quantum physics (see for example the ontogenesis of Feynman diagrams (p70) as discussed in [Tufte 2006b]).

diagrams in science

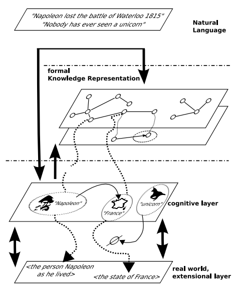
Besides the application of diagrams in the first steps of the scientific approach and as part of communication and teaching, Galileo Galilei first introduced them into the representation of final results in [Galilei 1613] which is praised in detail by Edward Tufte [Tufte 1990]. Since Galileo, diagrams have become a central tool of science which, on the one hand, have raised the quality of the publications, and on the other hand, have opened the door for simple manipulation of the reader via heavily biased diagrams accompanying "significant" statistic data [Beck-Bornholdt & Dubben 1999].

7.2.4 Diagrams and the Semantic Gap



(fig. 2.9 (excerpt))

The definition of abstract logical diagrams via a formal language approach and consequently the utilization of formal syntax and semantics allows to take a step analogous to the one from common language to logic (cf. fig. 2.9). Thus, two areas are entered simultaneously: the formalization of different semantic approaches together with their implementation in a computer science sense, as well as the difference between formal semantics and the pragmatic meaning, e.g., the intention of the conceptual modeller.



(fig. 2.11)

Consequently, the discussion of the gap between formal semantics and the intended semantics or between a formal and a cognitive foundation can easily be transferred to abstract logical diagrams. Regarding diagrams in general or even pictures, the inherent semantics cannot neglect perceptional features (cf. pictorial semantics at p76). Hence, the previously introduced semantic framework for linear languages (p32) needs to be extended to include this inherent perception and the differentiation between purely symbolic labelling relations and those in which resemblance cannot be factored out. Thus, classical linear language becomes a special case with strictly symbolically based labelling.

7.2.5 Why to prefer Diagrams (in certain Situations)

As formal diagrammatic languages can easily be translated to classical linear ones, there is, at least from the point of expressiveness and the underlying formal semantics, no advantage in utilizing abstract logical diagrams. Nevertheless, there are certain aspects that favour diagrammatic representations in certain pragmatic situations.

Diagrams are close to an engineer's first perceptive sketch of a situation (hypothesis 7) and, hence, the translation of this first informal, sketchy model into diagrams would be easier than a direct formalization into highly formalized notions like FOL. Again, this depends on the experience of the engineer with formal languages. But especially when communicating models (between human agents), diagrams are more easy to read. This is due to the usage of free rides which are known to the reader at a first glance because of the "closeness" to his own perceptive experiences and which do not have to be introduced as a part of a formal language, e.g., as predicates or basic relations. Consequently, this results in an intuitive understanding of diagrams especially in the context of knowledge transfer. This intuitiveness depends on the skills of the author of the diagram who has to have the knowledge to reduce the information to this intuitive level. The size of the depicted domain also plays an important role: at a certain size, graphical representations of a large number of entities and their relations are inferior to an ordered list representation.

The semantic gap still remains with formal diagrammatic languages. Nevertheless, visual languages allow to include iconic presentations of the depicted entities, e.g., the usage of prototype cards in the modelling game above, which are closer to perception and are a first step to minimize the semantic gap from the starting point of the formal language.

8 Sketch of an Advanced Categorical Approach *

The main problem of the previous FCA modelling attempt (sect. 6.3) was the missing formal foundation of the applied attributes, i.e., procedures to decide whether a property inheres in an object or not. In order to make the implicit interrelations of these properties which were derived from different theoretical approaches, explicit, these have to be reformulated in a stricter notion and based on the same underlying foundation.

basic idea

A new, basic approach has to integrate a model of perception and cognition as well as of semiotic denotation. On this basis, most properties of sect. 6.2 could be reformulated. Further, the implicit relations would become obvious because two properties have to be introduced on the same axiomatic foundation and consequently could be compared. The idea of a formal core ontology (p48) would propose such a basic fundament. Nevertheless, most fundamental issues cannot be solved with the help of a static description of entities but demand a model of the "social life" of these entities, i.e., their development over time and their quick interaction with others. There are different examples for the demand of dynamics: the preservation of the structures of real world objects under transformation to percepts or concepts; further, the construction of mental entities from basic building blocks; and the comparison of a simple semantic denotation relation to the outcome of the underlying cognitive algorithm as both match real world objects to (internal) concepts.

social life

[1] [Goguen 1991] introduced most of the basic modelling ideas behind category theory; for an introduction to this theory's basic concepts, see the references in sect. 10.1;

categories and related notions

The categorical approach of section 1.8 already tried to solve parts of this puzzle with the help of category theory and the theory of artificial perceptions. Besides the reasons to use category theory as a mathematical basic notion for cognition which was already discussed in sect. 1.8.2 based on [Macnamara 1994b], the previous chapters about pictorial representation allow to draw additional advantages.

[2] [Lawvere 1963] introduced the relationship between syntax and semantics of algebra-based formal languages;

A categorical notion would focus on structure-preserving relations (natural morphisms[1]) between real world objects, percepts and concepts. Further, compositional semantics[2] can be described by isomorphisms between syntax and semantics which, in the case of a semantics that is based on resemblance, can be transferred to the underlying real world objects. Differentiating between semantic notions demands for addressing the different mappings as objects (reification via arrow categories), e.g., to describe changes of the semantic labelling relation. The notion of commutative diagrams, which are tightly interconnected with categories, could form the base for a different definition of abstract logical diagrams that are directly based on a categorical semantics and allow for visual proofs by diagram chasing.

sketch of the model

The next figure (fig. 8.1) will give a first overview of a categorical approach which includes formal ontological considerations, e.g., a taxonomy of real objects regarding their utilization as a sign, with category theory. The goal of this approach would be a more rigorous reformulation of part I and part II of this thesis.

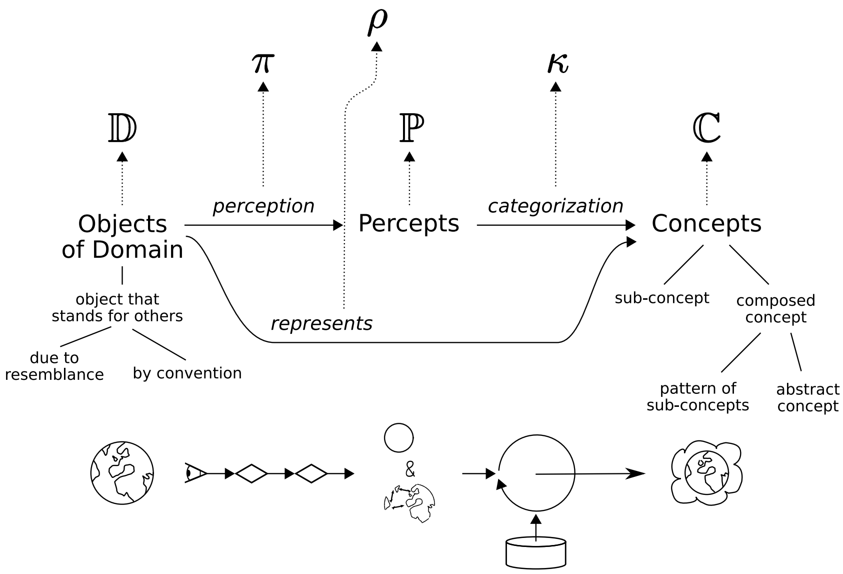


Fig. 8.1: Sketch of the Categorical Approach

$\mathbb{D}$, $\mathbb{P}$, and $\mathbb{C}$ describe categories of domain objects, percepts, and concepts. The next step would be the introduction of mappings between these categories, and, based on these, new categories that describe morphisms between mappings, e.g., like the p-morphisms between mappings of (external) objects to concepts in the category of artificial perceptions (def. 1.9). The three main types of mappings are embraced by the categories of perceptions π and the successive translation to concepts as the conceptualization κ; these are contrasted to direct mappings from objects to concepts with the help of the category of (symbolic) representation ρ. Fig. 8.1 explains the connection of these categories to the underlying cognitive model: the category π describes the filtering and extraction of Gestalt patterns, whereas κ portraits the search algorithm's mapping of basic patterns (percepts) to concepts. ρ would describe a relation between domain objects and concepts analogously to semantics.

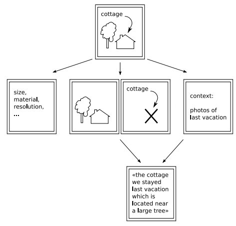
The simple taxonomy of concepts regarding their relation to perception (viz preliminary definition of abstractness (def. 6.1) and the lower part of fig. 8.1) would be reformulated with the help of these basic categories. For example, sub-concepts (def. 7) will be specified with the help of a structure preserving dependence (natural morphism) of a concrete mapping $k \in \kappa$ on an underlying decomposition into percepts $p \in \pi$. Similarly, most questions of semiotics can be reformulated with the help of categories, e.g., the equation $\rho^{-1}(\rho(d)) \stackrel{?}{=} d$ with $d \in \mathbb{D}$ recounts the question whether a symbol d stands for one concept only ($=$-case) which is equal to the basic requirement of notation systems (def. 9) to be one-to-one.

Hence, this formal notion would give a fundament to the considerations of the previous investigations and would solve most of the open problems introduced earlier, e.g., the proof of correctness for the cognitive algorithm (cf. sect. 1.5) or formalized properties as the basis for the modelling game like abstractness and metaphors (which are nothing else than special transformations between π and κ).

Nevertheless, this would include a beginning from scratch or – in the language of the following example part – another cycle in the circulus creativus. Hence, this will be left to future research (cf. sect. 16.3 for first ideas).

9 Outlook: A GFO coined View onto Pictures **

As will be mentioned in appendix D, GFO currently lacks a notion of semiotic entities; however, first drafts to include the entities «concept» as well as «information» are under discussion. Since pictorial presentation can be considered as an extension of the simple semiotic case there is currently no default way to include these entities. The strong connection of pictures to cognition, psychological entities like percepts, perception, and the bridging from personal perception of semiotic entities to culturally embedded language usage, all (post. 1), of which are indispensable for pictorial representation, underpin the importance of these entities in the ongoing discussions of the GFO community.



(fig. 5.1)

To conclude, pictures cannot be integrated into GFO directly but depend on the forthcoming capturing of the psychological, social, and mental stratum$^{\rightarrow}$.

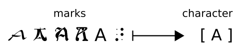
Regarding the theoretical approaches towards pictorial representation that were introduced earlier this part, there are several ways of approaching pictures with the help of GFO. The simple partition of a picture into an agglomerate of objects (cf. fig. 5.1) will be the starting point of the following approach.

Following postulate 8, a picture, i.e., its carrier, is a relatively lasting entity and will be modelled as GFO (material)[1] presential. (Regarding a dynamic approach to pictures which focusses on the act of creating a picture as the important entity, GFO allows to utilize situoids instead of material structures; nevertheless, this point of view was not elaborated in the previous analyses.) Further, the surface of the carrier that is perceived as a plane and that "carries" the Gestalt and symbolic content can be described by another material presential. This drawing plane presential depicts Gestalt-semiotic entities in a special configuration and in the case of diagrammatic representations has a spatial dimension of two. Hence, a novel symbolic relation «depicts» is demanded that relates signs (mental stratum) with their physical origin (cf. fig. 2.7 and the connection of marks to characters).

carrier presential

drawing plane presential

«depicts»



The context of a picture is specified with the help of a context situoid, i.e., an aggregation of facts of background knowledge.

context situoid

[1] GFO describes these presentials as «material structures»; in the following, the term "material presential" will be used to emphasize the presential character of these entities;

Regarding the ideas presented in sect. 6.2.2, the semantic content of a picture cannot be grasped by a single object of denotation. However, GFO supports a variety of possible basic categorizations for the entity that represents the meaning of a picture. Following Jorge Gracia, meaning is described by "*universals of a special form*"[2] [Gracia 1999]; this will be accomplished with the help of basic GFO entities in the following.

[2] "*. . . which represent a categorical symbol regarding an associated individual token*";

semantic situoid

Regarding simple semantic denotation, e.g., [-sense,-constr] (p34), a picture labels either another object, i.e., a material presential, (viz a prototypical picture of a horse), a process (flow chart), or a situoid (weather chart). As situoids can be projected onto their participating entities (material structures, processes, etc.) and their relation to space and time, the semantic content will be represented by a semantic situoid, e.g., the picture of a globe depicts a situoid which focusses mainly on one participating material structure – a single globe; this is described by a pragmatic focussing relation which is forgetful, i.e., ignores several aspects of the situoid in order to derive a semantic entity.

facts, propositions

Following the Wittgensteinian approach (p77), the semantic content of pictures can also be represented by facts and (predicative) propositions [Herre et al. 2006, p34f], i.e., related entities together with their relation.

Consequently, the entity that represents the semantic content of a picture can be formalized from different points of view regarding the underlying semantic theory. Further, the idea of the algorithm as part of the cognitive model could be translated to the decision for one of the possible meaning bearing entities from a pool of possible semantic objects[3]. For example, the possible meaning of the picture  could be described as the presential of a globe either as an individuum (a certain globe) or as a category (the class of all globes) as well as the proposition that a globe exists.

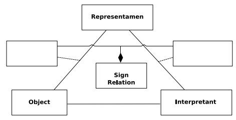
[3] cf. the choice between different semantic approaches to pictures regarding the background knowledge (p80);

pragmatic function

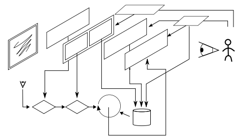
As pictures are used as part of communicative acts (post. 8), they are related to a pragmatic function. In GFO, a function is an "*intentional entity, defined in purely teleological terms by the specification of a goal, requirements, and a functional item [..] [they are] agent-dependent entities that primarily belong to the mental and social strata*" [Herre et al. 2006, p40]. Hence, functions can be utilized to describe the author's intentions as well as the requirements of the pragmatic context, e.g., their function in pedagogical knowledge transfer. Patryk Burek proposed a detailed approach to GFO functions which allows for utilizing functions in a normative sense, i.e., to describe categorizations of the picture domain which are based on a pragmatic context with the help of functional requirements [Burek 2007].[4] Further, functions can be tied to a process which describes the genesis of a picture and the resulting causal semantics (sect. 17), e.g., a photography depicts a certain object due to the underlying, photochemical causal process.

[4] a purely functional view onto pictures is described in [Doelker 2001] which could be axiomatized with the help of Burek's work;

Finally, GFO's subtle distinctions regarding relations, relators, and relational roles could form the basis for a more detailed analysis of Peirce's meaning triangle (cf. sect. 2.1.1); a more detailed view onto relational modelling will be presented in sect. 13.3. The meaning triangle is only a simplification of the relations that were presented as the meta-model of the picture domain in fig. 5.2. This meta-model would be the starting point for a more detailed GFO attempt to pictorial presentations compared to the above simple model which approaches pictures as a simple compound of five objects. Extending the idea of a sign as a relational role in a triadic semiotic relation (sect. 2.1.1), the meta-model's layers can be regarded as (relational) roles of the underlying relational structure. The different layers could be occupied by entities that are restricted by the previous considerations, e.g., a presential carrier plays the role of the physical layer.[5]



(extract of fig. 2.1)


(fig. 5.2)

[5] viz sect. 13.3 and the usage of player universals;

Nevertheless, the meta-model depends on entities of the social, psychological, and mental stratum which are still not fully elaborated in GFO. Hence, the modelling of the picture domain could be taken as the starting point and touchstone for the extension of GFO.

10 Synthesis

The previous chapters embedded and extend the frameworks of part I to result in the same final question: how to cross the gap between a *diagrammatic* formal languages semantics and the semantics intended by the modelling engineer?

Unfortunately, an all-embracing model of the diagram domain could not be proposed. Nevertheless, the meta-model (sect. 5.3) revealed the complexity that these general model has to cover; further, three concrete modelling attempts (the FCA game-based modelling (sect. 6.3), the axiomatic model (ch. 7), the sketch of the categorical model (ch. 8) resulted in inital conceptualizations that treat most important features of this domain and that allowed to transfer the results of part I on the semantic foundation of sentential formal modelling languages to the diagrammatic case.

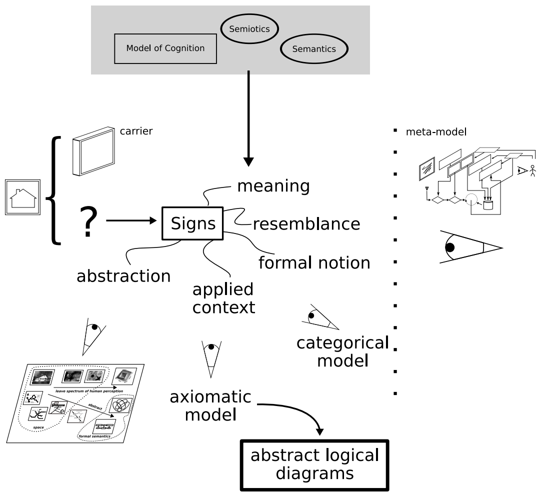


Fig. 10.1: Conceptual Map of Part II

The next part will enter the field of the semantic gap anew in the diagrammatic case by approaching the conceptual modelling of a simple domain with the help of a diagrammatic formal conceptual modelling language – in this case conceptual graphs.

10.1 Notes (Part II) $*$

Meta-Categorization

Alan Blackwell and Yuri Engelhardt also did a meta-categorization of approaches to diagrams [Blackwell & Engelhardt 2002] which additionally lists a a large number of other taxonomic approaches starting from diagrammatic research. As this thesis' starting point was the pictorial turn and thus pictures in general, these specialized approaches were mostly left out of discussion as they would not add new aspects to the more general pictorial theories. Nevertheless, this article's bibliography is a good starting point into the vast literature on diagrams.

[Doelker 2001] proposed a meta-model for pictorial representations that is based on the functional context, e.g., a class of pictures can be described as surrogates (i.e., their function is simulation).

Bildwissenschaft

Classical entries to Bildwissenschaft would include [Boehm 1994] or [Scholz 1991] starting from either an art-theoretic or a linguistic point of view.

The following three books have been central to most discussions of pictures since the 1960s: Nelson Goodman's "Languages of Art" [Goodman 1968], Gombrich's "Art and Illusion" [Gombrich 1960], and Arnheim's "Visual Thinking" [Arnheim 1969].

Besides his profound and elaborate approach towards pictures in [Sachs-Hombach 2006], Klaus Sachs-Hombach edited three central collections of articles in Bildwissenschaft: [Sachs-Hombach 1998b], [Sachs-Hombach & Rehkämper 2000], and [Sachs-Hombach 2001b]. Further, researchers of different domains explain their interest in Bildwissenschaft in [Sachs-Hombach 2005].

Diagrammatic Reasoning

Diagrammatic representation and reasoning is a central topic in knowledge representation and AI [Chandrasekaran et al. 1995] [Anderson et al. 2002]. Regarding the given entrée, these approaches mainly focus on a small domain of special diagrams, that share a certain closeness to representation and allow for geometric reasoning, e.g., free-body diagrams (p69).

Category Theory

Benjamin Pierce gives a short introduction to the most important principles of category theory while focussing on an exemplary application in computer science and avoiding a strong mathematical background [Pierce 1991]. A concrete use case of category theory in software engineering and the foundation of programming languages is presented by [Fiadeiro 2005].

Literature about the practical application of category theory is rare; one of the first articles was [Zimmer 1990]. Joseph Goguen summarized most of the basic underlying ideas in his "Categorical Manifesto" [Goguen 1991].

One of the best[1] guides into category theory are the introductory chapters of [Goldblatt 1986]. In the remaining book, Robert Goldblatt shows how to translate set theoretical constructs into categories and introduces the special category of "topoi"[2] that can be utilized as a fundament for a variety of logics, e.g., FOL and intuitionistic logics.

The "bible" of category theory is certainly [MacLane 1998] which anticipates readers with strong mathematical background.

The most influential publication that covers the utilization of category theory in cognitive science is [Macnamara 1994b] which includes a basic combination of logic and cognition [Macnamara 1994a] [Putnam 1994] [Magnan & Reyes 1994] as well as practical approaches [Lawvere 1994] and discussions of cognitive semantics [Bach 1994] [Pelletier 1994]. These different approaches are tied together by the underlying paradigmatic shift of cognitive science's research towards category theory.

[1] both mathematically profound and with enough examples to understand the concepts's background;

[2] topoi were originally introduced by Lawvere, see for example his introduction in [Lawvere 1994];

Part III

Conceptual Graphs:

Semantic Foundation and Interplay with Conceptual Modelling

Conceptual Graphs: From an Example of Part II's Framework to the Semantic Gap

As the antecedent introduction into the domain of diagrammatic formal languages lacked a basic example, this part will introduce conceptual graphs (CGs) as formal notion. After an example of practical conceptual modelling, the interplay of (diagrammatic) modelling languages and their semantic foundation will get into the focus as well as further ways to employ graphical notations in ontological and ontology engineering.

Based on a simple introduction of the ideas behind the notion of CG, a stricter mathematization based on [Dau 2003] will be presented which employs the relational model of formal concept analysis (FCA, see appendix C) as basic semantic structure. Besides this elaborated approach, other approaches for CGs's semantic foundation will be explicated briefly with special regard to part II's abstract view of diagrammatic semiotics.

Comparing a formal semantic formalization of relations with the CG model of a simple example domain, the drawbacks of the original paradigm will become obvious. As a result, a simple recipe for modelling relations with CGs will be proposed that feeds these considerations back into the graph framework. This modelling results in an enhancement of the standard CG framework which will be contrasted to extensions of the diagrammatic conceptual modelling language UML that originates from the same quandary.[3]

A short overview of possibilities to take advantage of CGs in the context of a formal ontology language like GFO will round off this last main part of the thesis.

[3] as UML will not be introduced, a basic knowledge is presupposed and those sections will be marked by "*";

11 Introduction to Conceptual Graphs

Without a standardized formalization of the idea of conceptual graphs, this chapter will outline the formal approach of Fritjhof Dau which is the most formally elaborated advance to the semantic foundation of these diagrams; additionally, other possible approaches will be briefly mentioned [Dau 2003]. Leaving aside some scarcely applied graphic features of this language, the following two chapters will introduce conceptual graphs as the diagrammatic formal language starting from a simple understanding of the graphical notion.

11.1 Origins and Neighbourhood

Conceptual Graphs (CGs) have their origin in Tesnière's Dependency Graphs [Tesnière 1959] and the Existential Graphs (EGs) of Charles Sanders Peirce [CP, 4.418–529][1] as well as the paradigm of entity-relationship modelling (ER) [Chen 1976]. Similarly to Peirce's EGs, CGs have the expressive power of FOL (CGs: [Dau 2003]; EGs: [Roberts 1992] [Zeman 1964]) and a graphical deduction system based solely on diagrammatic reasoning rules (cf. appendix E). Moreover, the graphical notion of CG gave raise to different *mathematizations*, i.e., stricter formulations in the sense of formal languages (cf. p39 and [Burstall & Goguen 1977]), and, in addition, became part of a capacious conceptual modelling paradigm [Sowa 1984]. Nowadays, with respect to their mutual interaction, the CG and the FCA paradigms have been combined into *Conceptual Structures* [Wille 1997].

[1] an excellent overview and introduction to EGs can be found in [Dau 2006];

mathematizations

Conceptual Structures

The next sections will introduce CGs's graphical notion, its discrepancies, and its embedding into a conceptual modelling framework, before giving an exemplary mathematization and an overview of different ways of these graph's semantic foundation.

11.2 Simple Conceptual Graphs

From a graph theoretic point of view, a CG remains nothing more than the graphical representation of a finite, directed, bipartite, not necessarily connected multigraph$^{\rightarrow}$.

finite, (directed,) bipartite graph

Therefore, the vocabulary of graph theory seems appropriate to describe the diagrammatic notion of CGs.

The vertices of the graph are partitioned into *concept nodes* and *relation nodes* which, in the following, are interpreted as concepts and the relations among them. Concepts are represented by their categorial type and a referent [TYPE : referent]; these are connected via relation nodes →(relation)→. The edges of the graph are directed arcs that represent the argument a concept occupies relative to a relation: the arrows lead from the first argument via the relation node to the second argument. Alternatively and with respect to relations with arity greater than two, the position of the associated concepts in the relation's argument are represented by an *edge labelling* –1–(relation)–2– which, furthermore, would allow to omit directed edges. As binary relations are used more frequently than those of other arities[2] and as the arrowed arcs of directed edges can be read intuitively[3], the first notion will be preferred. The following example will introduce most of the remaining graphical notions.

concept vs. relation nodes

[2] cf. discussion on CG mailinglist starting with `http://permalink.gmane.org/gmane.comp.ai.conceptual-graphs/2140`;

edge labelling

[3] this intuitive readability which is based on visual free rides is the reason for the popularity of arrow-and-node diagrams (p85);

Example 14 : An Advanced Graph

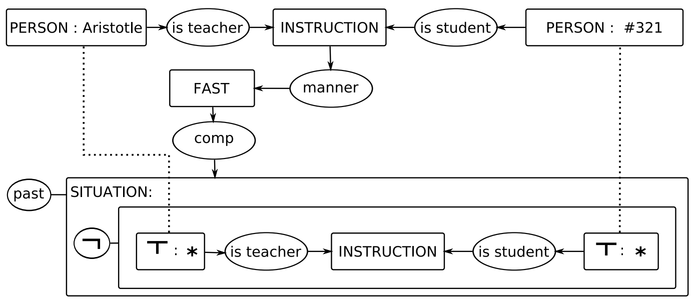


Fig. 11.1: An Example Conceptual Graph

The upper part of the example CG can be read prima facie: it is about an instructional situation with two participants playing different roles – modelled via relations – and, additionally, this teaching has a property that is explained in detail in the nested graph below.

Leaving aside for a moment the new graphical notions, the graph exemplifies different possibilities to give the referent of a concept. First, one can omit the referent [INSTRUCTION] to merely state that there exists a concept of that type (∃-quantifier: "there exists an instruction"). Second, a universal quantification (∀) is either applied via ∗ or can be restricted to a list of referents as applied in [⊤: ∗] where the ⊤ type combined with the ∗ referent is to be read as "any concept" or "something". Third, a concept can be referred to by a name, either a literal one like a real name [PERSON : Aristotle] or a locator like a (data-base) Id [PERSON : #321]. Fourth, one can refer to a concept by a description

modes of reference (expressing denotation) that is given by a nested CG as in the lower section of fig. 11.1; a concept whose designator is a non-blank CG is called *context*. Fifth, a *coreference* allows to refer to already introduced concepts. This is either achieved by EG's *line of identity* ·····, as in fig. 11.1, or equivalently by the usage of variables: in the concept $\boxed{\top : ?x}$ the variable x refers ("?") to the original concept $\boxed{\text{PERSON : Aristotle } *x}$ marked with the anchor $*x$. Further, in order to avoid formal problems due to semantics's sketchiness, section 11.4.1 will suggest to model coreference as a special CG relation $\doteq$ (see there).

context
coreference
line of identity

reference by variables

Example 14 : (continued)

Now the CG of fig. 11.1 can be translated to the subsequent statement: there exists an instruction with the person Aristotle as the teacher and the student person referred to by Id #321; this instruction has the property of being fast(er) compared (comp) to a situation in the past (past) which is described by a negated (¬) graph which represents another instructive situation between something (universal general type) related to it as the teacher and some student, whereas these related concepts are the same (line of identity) as above. More literally, this graph depicts that the person with id #311, e.g., Alexander the Great, is learning faster with the teacher Aristotle than before when he was not his student.

The correctness of this compositional translation or semantic interpretation, respectively, depends on a formal foundation of the CG language and a basic formal ontology. Regarding the definition of formal languages (def. 2.8), a formal syntax has to be fixated before stating a formal semantics; this syntax will be elaborated in the next section but one with the help of a mathematization.

11.3 The Conceptual Graphs Framework

Based on example 14, other important techniques of the CG framework are to be exposed. Reconsidering concepts, their types can be related by a subsumption hierarchy including $\top$ and $\bot$ as the general type and absurdity. This taxonomy, often labelled (CG-) *ontology*, accompanies each CG and plays a central role in reasoning. Besides this implicit reasoning, the origin of CGs in EGs allows to transfer their graphical deduction rules with little effort (see extensive example in appendix E).

ontology

Presupposing the idea of the axiomatic deductive method (def. 2.12), the techniques of conceptual abstraction and relation contraction are the basic building blocks of definition. *Conceptual abstraction* [Sowa 1984, Def. 6.3.1, p104] allows the definition of new concepts on top of already articulated ones. Analogously, *relation contrac-*

conceptual abstraction & relation contraction

tion defines relations [Sowa 1984, Def. 3.6.12, p114]; both will be introduced in the context of the previous example.

Example 14 : (continued)

This example's conceptual graph utilizes the concepts and relations without antecedent definition; in correspondence with the axiomatic deductive method, each is to be understood as an axiom whose meaning is given by common sense. [Sowa 1984, appendix B] introduced first concise lexical definition of the standard CG entities based on a relatively simple ontology which was elaborated in [Sowa 2000]. Assuming the example's entities as already predefined and without formally introducing conceptual abstraction and relational contraction, the figures 11.2 and 11.3 should be easily readable.

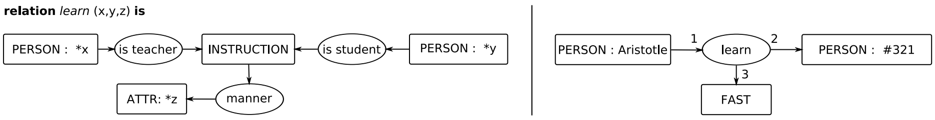


Fig. 11.2: A Sample Relational Contraction and its Result

Fig. 11.2 introduces a new ternary relation «learn» between two persons and an attribute: fig. 11.3 defines a new concept «teacher» as a person that is in the «is teacher» relation towards some instruction (∃-quantification) and that is not equal to the student.[4] The notions which are introduced in these figures expose another important application of reference by variables: they allow to refer to CG entities from outside the graph.

[4] the line of identity is substituted by the ≐ relation and the negation by a cut which, due to section 11.4.1, are the formal correct expressions in this case, as will be shown later;

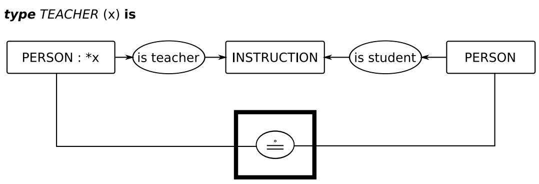


Fig. 11.3: Simple Conceptual Abstraction

Prudently, the above examples have a flaw: without applying the axiomatic deductive method from the start, the relation «is teacher» seems dependent on an understanding of teacher which should be made explicit in the definition of the concept «teacher». Avoiding circular definitions, «is teacher» is meant to explicate the role played by a person in the instruction. (Relational) Roles and their modelling in CGs will be part of the next but one chapter's analysis.

11.4 Simple Concept Graphs with Cut

This section introduces *simple concept graphs with cut* which, on the one hand, serve as a mathematization and formal stricter version of CGs. On the other hand, the following procedure can be seen as a further example of the foundation of a formal diagrammatic language (cf. sect. 7.2). This whole section is adapted from Fritjhof Dau [Dau 2003].

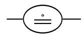
11.4.1 Problems with CGS

Due to a formal language's underlying principle of compositionality, the assumed inseparability of syntax and semantics (p83) is a distinctive feature of diagrams. Therefore, before introducing one possible formal syntax for conceptual graphs, the most prominent semantic difficulties which through this backdoor influence the definition of the formal language are to be brought to light.

Following [Dau 2003, p14, p187ff], problems arise due to the unclear semantic status of negation, the usage of coreference, the status of nested graphs, and the interplay of these phenomena.

Hence, Dau's design decisions which try to avoid these problems will be adopted: first, negation is not expressed via an unary relation but by *cuts* reminiscing the EG origin; cuts are highlighted areas in the diagram (here: rectangles with bold lines) whose content has to be interpreted semantically as negated; second, instead of the above approaches to depict coreference (variables and lines of identity), the identity of two concepts is modelled via a special identity-relation ($\doteq$). This results in graphs like fig. 11.4 which can now be read easily as "there are two things that are not (cut) identical" therefore "there exists more than one thing".

cuts



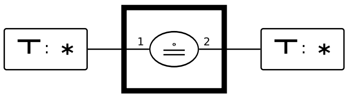


Fig. 11.4: Simple Concept Graph with Cut

11.4.2 Basic Definitions

As semantics precedes syntax (cf. p83) and [Dau 2003] progresses towards a formal semantics based on FCA, the following definitions, which will result in existential simple concept graphs with cuts, intrinsically include a basic understanding of FCA as introduced in appendix C. First, a basic relational structure is defined.

Definition 11.1

relational graph with cuts

A *relational graph with cuts* is a sextuplet $(V, E, \nu, \top, Cut, area)$ with pairwise disjoint finite sets V, E, and Cut for vertices, edges and cuts. The bijection $\nu : E \overset{\text{bij.}}{\Leftrightarrow} \bigcup_{k \in \mathbb{N}} V^k$ relates edges to vertices; further, for $e \in E$ define $|e| = k$ s.t. k is the corresponding size of the tuple of vertices related to e by ν.

The most fundamental graph is the ‘empty graph’ called *sheet of assertion* $\top \notin V \cup E \cup Cut$.[5]

sheet of assertion

[5] the “tabula rasa” (p70) of CG;

area relates the area of a cut to its contents via the mapping

$$area : Cut \cup \top \to Pow(V \cup E \cup Cut)$$

such that

- if $c_1 \neq c_2$ then $area(c_1) \cap area(c_2) = \emptyset$ for $c_1, c_2 \in Cut \cup \top$
- $V \cup E \cup Cut = \bigcup_{d \in Cut \cup \{\top\}} area(d)$
- $c \notin area^n(c)$ for each $c \in Cut \cup \{\top\}$ and $n \in \mathbb{N}$
  (with $area^0(c) := \{c\}$ and $area^{n+1}(c) := \bigcup\{area(d) | d \in area^n(c)\}$)

A context c may contain other contexts in its area, and therefore induces a tree-like order on contexts with root $\top$ where $c \leq d$ means “is deeper nested”[6]. Hence, the set of all contexts can be traversed root-down in a breath-first fashion which will be used later when evaluating a graph’s variables.

[6] cf. fig. 12.4 which shows this orders importance to semantic evaluation;

To sum up, relational graphs with cuts resemble graph theoretic graphs with relations of arbitrary arity. The additionally introduced cuts superimpose the structure of the graph such that they include concepts whereas relations can traverse their borders; in addition, they form a nested hierarchy, i.e., they are only properly included in other cuts or the sheet of assertion.

Regarding the standard procedure to introduce an algebraic formal language, in the next step, a signature has to be defined.

Definition 11.2

alphabet (signature)

Let $Var := \{x_1, x_2, \ldots\}$ be a countably infinite set of signs with $*$ as the generic marker. For each variable $\alpha \in Var$ a new sign $*_\alpha$ is assigned.

An Alphabet is a triple $\mathcal{A} = (\mathcal{G}, C, \mathcal{R})$ of disjoint sets $\mathcal{G}, C, \mathcal{R}$ such that

$\mathcal{A} = (\mathcal{G}, C, \mathcal{R})$

- $\mathcal{G}$ is a finite set (names of “Gegenstände”, cf. appendix C)
- $(C, \leq_C)$ is a finite ordered set with greatest element $\top_C$
- $(\mathcal{R}, \leq_\mathcal{R})$ is a family of finite ordered sets $((R)_k, \leq_{\mathcal{R}_k})_{k=1,\ldots n}$;
  $\doteq \in \mathcal{R}_2$ is a special relation name which is called identity

Now, one defines an order on $\mathcal{G} \cup \{*\}$ with greatest element $*$ and all other elements of $\mathcal{G}$ are incomparable.

Te order on G allows to subsume each name under the generic marker, i.e., the generic marker can be used as their representant. Concept graphs can be introduced as follows:

Definition 11.3

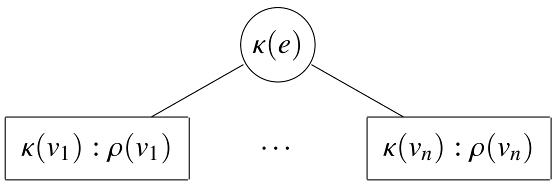
(existential) simple concept graphs with cuts

A *simple concept graph with cuts* and variables over the alphabet $\mathcal{A}$ is a structure $\mathfrak{G} := (V, E, \nu, \top, Cut, area, \kappa, \rho)$ where

- $(V, E, \nu, \top, Cut, area)$ is a relational graph
- $\kappa : V \cup E \to \mathcal{C} \cup \mathcal{R}$ is a mapping such that
  - $\kappa(V) \subseteq \mathcal{C}$, $\kappa(E) \subseteq \mathcal{R}$ and
  - all $e \in E$ with $|e| = k$ satisfy $\kappa(e) \in \mathcal{R}_k$
- $\rho : V \to \mathcal{G} \cup \{*\} \cup \{*_\alpha : \alpha \in Var\}$ is a mapping

Draw Concept Graphs

A view onto the graphical notion reveals the simplicity of the above definition. κ and ρ map a vertex v onto its type and referent, i.e., concept and relation names. The important feature is the usage of concept *names*, not concepts. The relation between these will be laid down when defining semantics on top of this whole approach in section 12.4. Hence, an edge $e = (v_1, \ldots, v_n)$ is depicted by



The next chapter will give a summary of different ways to state a semantic foundation of CGs, whereby only Dau's approach will be explicated in detail.

12 Overview of Approaches to CG's Semantic Foundation

Classically, the consecutive step to introducing a formal language's syntax is the semantic foundation in (relational) models either by translation to a formal language that already has a semantic foundation like FOL or by an original approach in the light of the semantic framework of sect. 2.4 and its extension to diagrammatic languages in sect. 7.2.

12.1 Sowa's original Approach and Common Logic

Originally, John Sowa proposed to translate a conceptual graph to a formula of FOL and consequently use FOL models, i.e., relational structures as exposed in appendix B, for CGS's semantics [Sowa 1984]. This approach had its prequel in the usage of Hintikka's surface models as semantics for CGS [Sowa 1979]. Sowa suggested a translation operator Φ which, as Michel Wermelinger discovered formal lacks in the original definition [Wermelinger 1995], was finally formally revised by Dau [Dau 2003, p97]. This translation was nevertheless based on Peirce's idea behind EGS as an easily readable notion of FOL formulae. The next example will introduce this basic translation from CG to FOL with the help of Φ.

Example 15 : From CG to FOL

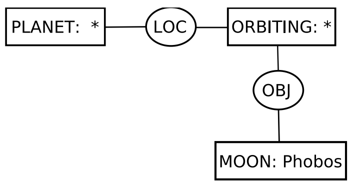


Fig. 12.1: Point of Departure of Translation Operation

First, the operator Φ is to be specified due to table 12.2 which (informally) introduces the mappings needed to translate the graph of fig. 12.1.

CG :	FOL :
generic concept	distinct variable symbol
concept	monadic predicate whose name is $type(u)$ and argument is its variable (generic) or given referent
conceptual relation	n-adic predicate whose i-th argument is concept related to i-th arc
negative context c	$\Phi(c) = \neg p$ with p proposition linked to c
⋮	⋮

Fig. 12.2:
Partial definition of translation operator Φ

The resulting formula can be written as

$$\begin{aligned}\exists x_1\, \exists x_2 \quad : \quad & ORBITING(x_1) \wedge PLANET(x_2) \\ & \wedge MOON(Phobos) \\ & \wedge LOC(x_1, x_2) \wedge OBJ(x_1, Phobos)\end{aligned}$$

Evidently, the resulting formulae of Φ are in existential normal form, i.e., can be rewritten in the form

$$\underbrace{\exists x_1, \cdots, \exists x_n}_{head} : \bigwedge_{i=1}^{k} \underbrace{\phi_i}_{quantifier-free}$$

Common Logic

Nowadays, CG is a fully conformant dialect of Common Logic (CL) as standardized in [ISO/IEC JTC 1/SC 32 2006]. Hence, CGS are defined as a formal language CGIF and then translated to expressions in abstract CL syntax applying their relational structure semantics. The disadvantage[1] of these CL-CGS is the abandoning of a graphical notion as the starting point. Herewith, CGS are nothing more than a *linear* CL formula which has an additional graphical rendering. Therefore, these CGIF graphs will not be analyzed further. Another variance is the logical status of CL that is beyond FOPC: CL is not second order, hence still a first order logic, but allows expressions beyond FOPC that are translatable to standard FOPC expressions[2].

[1] disadvantage at least from the point of the pictorial turn's focus on purely diagrammatic languages;

CGIF

[2] see discussion at CG-mailing-list starting with `http://article.gmane.org/gmane.comp.ai.conceptual-graphs/1675` ; further, the CGIF extension IKL will go beyond simple FOPC [Hayes 2007];

12.2 An Extensional Graph Semantics

The French CG School[3] was the first to give an extensional semantics to CGs, i.e., a direct mapping to a relational structure [Chein & Mugnier 1992] [Chein & Mugnier 1995] [Chein & Mugnier 1996] [Mugnier 2000]. The idea behind is relatively simple: as already stated, CGs resemble the graphical representation of finite, undirected, bipartite, not necessarily connected multigraphs→. The specific feature of this approach resides in the usage of *graph homomorphisms*→ to describe CG reasoning. As most graph-homomorphisms have effective[4] algorithmic implementations, this reasoning is relatively fast (compared to the translation to FOL and FOL reasoning[5]); furthermore, the results of the reasoning become reproducible in a graph-way as opposed to the FOL way which only provides the final result, e.g., the question about the equality of two graphs is represented in fig. 12.3. (To avoid confusion: this framework only covers a subclass of CG which is decidable, hence, the usage of graph based reasoning is both decidable and polynomial.)

[3] as all these authors share a common understanding of CG, this title seems appropriate;

graph-homomorphisms

[4] to commemorate: effectiveness equals to a time-complexity in **P**;

[5] FOL / FOPC reasoning is undecidable in the general case; nevertheless practically "usable" implementations exist for subsets;

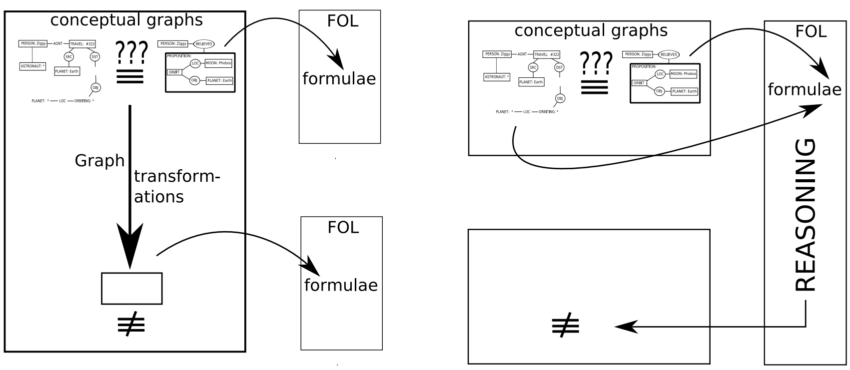


Fig. 12.3: Comparison of FOL- and Graph-based Reasoning

In the eyes of the French School [Chein & Mugnier 1996], this approach results in certain advantages over the classical translation method: first, an inference calculus based on graph morphisms allows the usage of fast graph algorithms; second, the graph theoretic notion is relatively close to the original graphical notation of CG; third, the semantics is consistent and simple [Mugnier 2000] as well as easily extensible (as an example: the original notion did not include nested graphs which were added in [Chein & Mugnier 1996]); and finally, these graphs exhibit a sound and complete reasoning [Salvat & Mugnier 1996].

nested graphs

12.3 Outlining Other Initial Approaches ∗

The three basic approaches to semantics are the extensional graph theoretic semantics, the translation to FOL or CL formulae, and their semantic interpretation (see above), as well as the extensional semantics based on FCA which will be presented in the next section in detail. Furthermore, other approaches were proposed which resemble the basic ideas of pictorial semantics as presented in section 6.2.2 (p76ff).

An Algebraic Approach

Algebra is one of the cornerstones of all mathematical modelling and was already applied, because relational structures, e.g., for the semantic foundation of FOL or FCA, are nothing more than *algebras*, i.e., sets with certain operations and additional side conditions. Besides, most entities of a programming language, e.g., abstract data types, as well as mathematical entities like lattices are trivially (universal) algebras.

algebra

[Bräuner et al. 1999] enhances the distributive lattice of CG-concepts with a binary relational algebra which leads to a two-sorted algebraic logic with two sorts of concepts which are modelled by unary predicates and relations. With the Peirce-product

write CG as term of 2-sorted algebraic logic

$$[\![r : c]\!] = \{x \mid \exists y \in [\![c]\!] : < x, y > \in [\![r]\!]\}$$

one can simply map a conceptual graph to the term $c \times (r_1 : c_1) \times \cdots \times (r_n : c_n)$ and therefore, using additional axioms, apply deductive reasoning on CGs by term-rewriting.

reasoning by term rewriting

Discourse Representation Theory

Discourse representation theory [Kamp et al. 2003][6] is an extension to FOL which allows to model dynamic semantics. This approach emerged as a part of natural language semantics to counterpart the dependence of meaning on contexts which are themselves subject to change.

[6] see [van Eijck 2005] for a shorter introduction;

The basic idea is to model contextual information with *discourse referents* such that each discourse, i.e., a sequence of (natural language) sentences, is interpreted in a *discourse representation structure* (DRS). While interpreting, the DRS is updated, i.e., it dynamically adapts to the new information. The DRS is modelled via an intensional model of possible worlds with unique names, whereas relations are interpreted in a particular world, and worlds are generally interconnected.

discourse referents

DRS

intensional semantics

[Kerdiles 1999] translates a single CG directly to a discourse referent in a DRS. Consequently, the meaning of the graph becomes the change of information with respect to the DRS. Further, this interrelation can be exploited for analogical reasoning on a CG-based knowledge base. The distinctive feature of this approach is the focus on dynamics and the usage of an intensional possible world semantics. On the one hand, this dynamics enhances the semantic vocabulary, i.e., what can be expressed

dynamic semantics

with CGs; on the other hand, DRs complicate the definition of the underlying labelling relation.

12.4 CGs and Formal Concept Analysis

[Klinger 2005, Appendix A.2] lists and compares a variety of possibilities to define concept graphs formally on top of FCA. Furthermore, Julia Klinger explicates different modes of utilizing these formal models for semantic foundation. The following approach connects the above definition of existential simple concept graphs with cut with FCA's power context families and, again, is taken from [Dau 2003]. The other listed approaches differ in the modelling of FCA relations (see appendix C), the handling of negation and cuts, as well as the evaluation of variables and the possibility to use nested graphs.

From Concept Graphs to Power Context Families

[Dau 2003] suggests to interpret existential simple concept graphs with cut in power context families[7]. Def. 11.3 did assign CG concepts and relations to (FCA like) concept and relation *names*. In a nutshell, object names are now mapped to objects, concept names are assigned to formal concepts of $\mathbb{K}_0$, and relation names of arity k to relation concepts of $\mathbb{K}_k$. Thence, the formal model of those concept graphs are contextual models which extend FCA's standard power context families (Def. C.3) with the suggested interpretation.[8]

[7] cf. appendix C for an introduction, especially of the notions of $\mathcal{G}, C, \mathcal{R}, \underline{\mathfrak{B}}, \Re, Ext$;

[8] again, the following definitions are taken from [Dau 2003];

Definition 12.1 — contextual models

$(\vec{\mathbb{K}}, \lambda)$ is a *contextual model* whereas $\vec{\mathbb{K}}$ is a power context family (cf. appendix def. C.3). The $\vec{\mathbb{K}}$-interpretation over the alphabet $\mathcal{A} = (\mathcal{G}, \mathcal{B}, \mathcal{R})$ can be decomposed into object names, cuts, and relations $\lambda := \lambda_{\mathcal{G}} \mathbin{\dot\cup} \lambda_C \mathbin{\dot\cup} \lambda_{\mathcal{R}}$ which are defined as follows:

- $\lambda_{\mathcal{G}} : \mathcal{G} \to G_0 \qquad \lambda_C : C \to \underline{\mathfrak{B}}(\mathbb{K}_0) \qquad \lambda_{\mathcal{R}} : \mathcal{R} \to \Re_{\vec{\mathbb{K}}}$
- λ_C and $\lambda_{\mathcal{R}}$ are order-preserving
- the top cut maps to the sheet of assertion: $\lambda_C(\top_C) = \top$
- relations are higher order concepts:
  $$\lambda_{\mathcal{R}} \subseteq \underline{\mathfrak{B}}(\mathbb{K}_k), \text{ for all } k = 1, \ldots, n$$
- conceptual identity is defined explicitly:
  $$(g_1, g_2) \in Ext(\lambda_{\mathcal{R}}(\doteq)) \Leftrightarrow g_1 = g_2 \text{ for all } g_1, g_2 \in G_0$$

The next step in defining a formal language is the introduction of an evaluation assignment $\models$ which is always presented in an inductive manner. In the following, evaluation heavily depends on the extensions of FCA concepts.

Definition 12.2

$\models$ (propositional part)

An evaluation $(\vec{\mathbb{K}}, \lambda) \models \mathfrak{G}[c]$ ($c \in Cut \cup \{\top\}$) is defined inductively as:

- vertex condition:
$$\lambda_{\mathcal{G}}(\rho(v)) \in Ext(\lambda_{\mathcal{C}}(\kappa(v))) \text{ f.e. } v \in V \cap area(c)$$
- edge condition:
$$\lambda_{\mathcal{G}}(\rho(e)) \in Ext(\lambda_{\mathcal{R}}(\kappa(e))) \text{ f.e. } e \in E \cap area(c)$$
- cut condition (iteration over $Cut \cup \{\top\}$):
$$(\vec{\mathbb{K}}, \lambda) \not\models \mathfrak{G}[c'] \text{ f.e. } c' \in Cut \cap area(c)$$

The goal of an evaluation is an interpretation $(\vec{\mathbb{K}}, \lambda) \models \mathfrak{G}[\top]$ for the entire graph which is approached as the content of the sheet of assertion $\mathfrak{G}[\top]$. However, the usage of variables (e.g., for coreference) needs a special treatment. The variables must be mapped to formal concepts by a *valuation* $ref : V' \to G_0$.

valuation

Definition 12.3

A valuation $ref : V' \to G_0$ is *partial* :iff

partial valuation

$$V^{\mathcal{G}} \subseteq V' \subseteq V \;\&\; ref(v) = \lambda_{\mathcal{G}}(\rho(v)) \text{ f.a. } v \in V^{\mathcal{G}}$$

and *total* :iff

total valuation

$$\{v \in V^* | v > c\} \subseteq V' \;\&\; V' \cap \{v \in V^* | v \leq c\} = \emptyset$$

endoporeutic vs. classical evaluation

A concept graph can be evaluated in two ways: the classical (FOL) way $\models_{class}$ and the endoporeutic method $\models_{endo}$ whose goal is to step-by-step generate a total evaluation of a concept graph from partial evaluations. The endoporeutic method was proposed by Peirce[9] and its outcome can easily proven to be equal to the classical Tarskian FOL evaluation of sect. 2.3.[10] This result can be extended to simple concept graphs [Dau 2003, ch. 11].

In the classical case, a total evaluation is given a priori and whenever an $\exists$-quantifier is evaluated, either directly in a FOL formula or as an $\exists$ statement in a concept graph, the quantified variable is substituted with the result of the mapping.

The endoporeutic method generates the evaluation "from the outside in", i.e., the formula or graph is read beginning with the outermost quantifier, i.e., the sheet of assertion, and working towards the deepest nested concepts. Whilst crossing deeper nested quantifiers or cuts, one successively assigns values to the quantified variables

[9] cf. CP 4.408 or [Sowa 2005]; as Roberts notes (due to [Pietarinen 2004]), Peirce did never use the term 'endoporeutic' in the context of EGs [Roberts 1973];

[10] [Hilpinen 1982] proves the equality of the endoporeutic method and Hintikka's game theoretic semantics which is to be known (simple proof by induction) to be equal to the classical FOL semantics;

or, in the case of graphs, the concepts contained in a cut that are connected to already assigned concepts via lines of identity. The following example shows a simple endoporeutic evaluation.

Example 16 : Endoporeutic Evaluation

The evaluation of the graph $\mathfrak{G}$ of fig. 12.4 starts from the sheet of assertion $\top$, therefore only cut c_1 is directly enclosed. Hence, $\mathfrak{G}$ is true if the part of it that is enclosed by c_1 is false[11]. Now, one proceeds to evaluate the graph inwardly or, regarding the tree-order of cuts, top-down and breath-first.

[11] remember: cuts model negation;

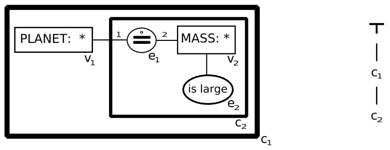


Fig. 12.4: Concept Graph to be Evaluated (with order of Cuts)

The cut c_1 is true if there exists an object o_1 such that o_1 is a «planet» and that the sub-graph enclosed by c_2 is false. Further on, c_2 is true if there is an object that is both a «large mass» and identical to o_1.

Collecting the steps of the evaluation above, the CG $\mathfrak{G}$ is true if there is *no* planet such there is no other object that is identical to it and that is a large mass, or in simple terms: every planet is a large mass.

From the above example, the meaning of double cuts (with $area(c_1) \cap V \neq \emptyset$) seems obvious: they are nothing more than material implications. Consecutive double cuts are nothing more than double negations which would be deletable by CG's graphical deduction (cf. appendix E). A trained reader of CG (and EG) would have read the graph instantaneously in this manner.

12.5 CGs as (Diagrammatic) Formal Language

To summarize this chapter, a formal language for CGs which, in the spirit of def. 2.8 and its extension def. 7.3, can be written as $(\mathcal{A}, syn, sem, \models, \vdash)$, was introduced via the signature $\mathcal{A}$ (def. 11.2) and the definition of concept graphs with cut (def. 11.3) as its basic syntactical structure *syn*. The semantics *sem* and the semantic entailment $\models$ were introduced via contextual models (def. 12.1), their interpretation (def. 12.2), and the endoporeutic evaluation method (cf. p126). To complete, a short glance onto the graphical deduction $\vdash$ is given in appendix E. Hence, conceptual graphs are abstract logical diagrams in the strong sense (def. 7.5).

The next step would be to analyze this language's (meta-)properties, which was rudimentary commenced in [Dau 2003] by proving the upper formal language to be sound and complete (compare to FOPC's meta-properties in appendix B). The quality of a modelling language can only be evaluated in a practical setup, i.e., its acquirement to express the aspects of a domain which an engineer wants to model. The next chapter will apply CG to catch the main features of an example domain: the domain of trust.

13 Modelling Relations with CGS: A Practical Example

Hitherto, the approach towards CGS was coined by formal language and formal semantics whereas the (semiotic) meaning of graphical entities was expressed by relational structures. Nevertheless, these graphs are widely used in knowledge representation (cf. p40) and conceptual modelling (def. 2.11), and are therefore applied to "describe aspects of the (real) world" (ibid.).

This chapter will return to the basic problem of sections 2.4 and 7.2.4: how to relate the (formal) semantics of formal languages with the meaning that is aspired by the modelling engineer. This will be made explicit with the help of the example of modelling a simple domain and, based on this, an investigation of the reciprocation between the semantic foundation of a modelling language and its practical usage. This interconnection is best depicted by the – at a first glance – vicious circle of fig. 13.1 which will be shown to be a *circulus creativus*[1] in the end.

[1] the "creative circle" is a term of von Foerster's epistemology [von Foerster 1985];

circulus creativus

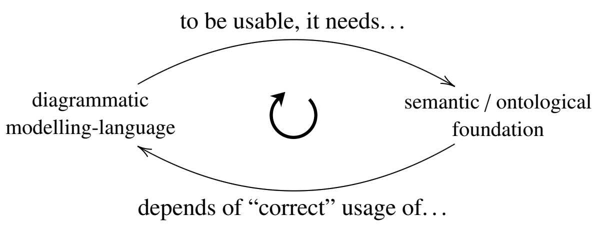


Fig. 13.1: Circulus Vitiosus or Circulus Creativus?

This chapter will take the following route: a simple example domain is approached by conceptual modelling with CGS; the resulting problems lead to an ontologically based analysis and the conclusion, that the notion of CG, as introduced in the previous chapter, needs to be extended to grasp the example domain. As the other important diagrammatic modelling language – UML – also fails to express the domain's subtle relations, different ways to expand UML are proposed which serves as inspiration for an extension of CGS to concept graphs with relators.

13.1 Introducing the Example Domain

To avoid confusion with the already mangled examples, the following example for CG relations will be the situation of trust as formalized by [Coleman 1990] and [Buskens 1999]. The domain will be presented by a prototypical situation and a generalized description mingled with a first – already slightly – formalized approach.

Example 17 : Trust

trust

Trust is a quaternary relation $trust(X, Y, S, A_G)$ between two social agents X and Y, which participate together in the contextual situation S. This situation involves an action that involves a good G belonging to X and which is currently at the disposal of Y. X trusts Y in the situation S to apply action A_G. Normally the action lies a certain amount of time in the future which accounts for the risk the trustor must take. The relational roles of X and Y are labelled «trustor» and «trustee» [2].

For example, this relation holds in the situation of lending a book. The two agents are the person lending the book, called lender, and the borrower who is trusted return the book (A_G) after a certain amount of time.

[2] avoiding the ontological discussion whether roles and concepts share the same type (see references in later chapters), role names will be written like concept names in guillemets which resembles their possible modelling as UML stereotypes;

13.2 A naïve CG Approach

The next graphs approach step-by-step a conceptual model of this domain whereby initial problems will emerge immediately.

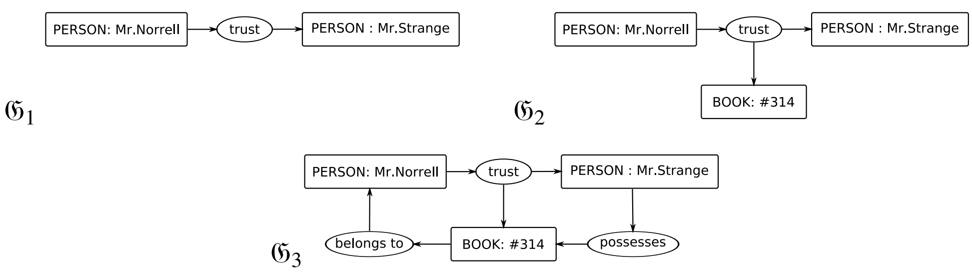


Fig. 13.2: A first Approach

Fig. 13.2 introduces graphs of proceeding complexity: starting from trust as a simple relation between two concrete persons ($\mathfrak{G}_1$), the object of trust and its relation to the two persons is introduced ($\mathfrak{G}_2$ and $\mathfrak{G}_3$). Leaving aside for a moment the modelling of the action and its embedding in time which would require advanced temporal modelling techniques, $\mathfrak{G}_3$ is lacking the assignment of the relational roles which describe the positions of the related persons towards the relation. Therefore, a more detailed analysis of relations and relational roles is necessary which will lead to the question whether the above graphs model the intended domain. Moreover, different ways to enhance the Cg framework will be introduced which will allow to easily express the desired properties of the domain.

13.3 Approaching Relations from Formal Ontology

> "*Relations are very peculiar entities; [...] [Many philosophers] have thought that relations are nothing other than the relata and their features or that they are merely appearances. But others have conceived relations as the very stuff from which the world is ultimately constituted.*
> *[...]*
> *Indeed the idea that metaphysics studies only relations is highly exclusionary unless one accepts the controversial view that the world is composed of nothing but relations. This view is controversial on various counts, two of which are quite evident: First, our experience seems to vouch for the existence of things other than relations, and second, the very notion of relation seems to presuppose the notions of non-relational entities, the relata which are tied by the relation. In short, this line of thought does not seem promising.*"
>
> [Gracia 1999, p58f]

These basic meditations lead back to the philosophical preliminaries of section 1.1 and are conform to the basic perceptional approach presented in chapter 1. Putting relations into the foreground of modelling, the postulate of objectivism (post. 2, p9) must be enhanced in the following manner:

Postulate 9 — objectivism (with relations)

> There is a realm of objects; this realm exists independently of one's mind. These objects are interrelated, i.e., relations are entities qua objects, but nevertheless dependent on the prior way of dissecting this realm into objects and the relations among them. Further, relations can be reified, i.e., analyzed with the same questions and formal tools as objects.

This approach is quite natural and superimposes most of the presented paradigms. The next subsection will exemplify the different roles played by relations regarding their level in an ontology.

13.3.1 Different (Meta-)Levels of Relations in CG

The idea of levels or layers emerges[3] which can be regarded as enhancing the analysis of the interrelation of different languages (p46) further to their corresponding model.

[3] see ontological stratum→ in glossary;

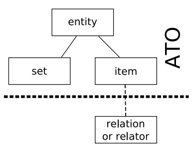
In the CG framework introduced above, different archetypes . of relations can be found: first, relations were introduced as the relation nodes that depict relations between the CG's concepts (see the previous chapter's translation via semantics to FCA concepts and power context families); second, these relations can be reified as (relation-) concepts, as will be introduced later in more detail, and therefore model a domain's relations with the power of CG concepts; third, the above definitions make use of mathematical relations, e.g., to relate a concept node with type and referent, which can be, due to their axiomatic deductive nature, traced back to set-theoretical relations; fourth, the arcs between concept and relation nodes are relations in the sense of the underlying graph theoretic structure and therefore also in the sense of the previous set-theoretic reduction. Leaving aside for a moment the different kinds of relations which these notions can map to, e.g., FCA's relation concepts, real world relations, or relations between percepts, these multifarious relations are woven tightly together and depend on each other[4].

[4] this interwoven multitude of concepts can also be seen with «concepts» itself: CG's concepts, FCA concepts, conceptual space's concepts, etc.; see detailed discussion at appendix C pconcepts-versus;

Nevertheless, from the standpoint of conceptual modelling, only the notions that represent the relations of the domain, pro re nata the relation and concept nodes, are of interest whereas the mathematical notions will be left aside. On top of this restriction, an already elaborated approach towards relations will be considered in the next chapter focussing on relations from a formal ontological standpoint.[5]

[5] the view onto diagrammatic relations as the images of real world relations could be contrasted to Deleuze's idea of diagrams as autopoietic structures (p79);

13.3.2 Relations in GFO



(fig. D.2 (excerpt))

GFO (for a short introduction see appendix D on p168) respects different levels of relations by segregating the set-theoretic basic relations from ontologic relations or relators (as subsumed under item) at the first possible point of distinction (cf. D.2) in its abstract top ontology (ATO).

In brief, *relations* "*bind [a finite number of] things of the real world together*" [Herre et al. 2006, p33]. These are the *relata* of the relation and their number is the *arity* of the relation. Moreover, the relata can play the same or different role in the context of the relation. Relations exhibit a categorial character, i.e., they generalize a kind of entities which form the "glue" among other entities; these *relators* are "*aggregate[s] of all the [qua individuals→] that share the same foundation*" [Guizzardi 2005, p240]. In other words, a relator is the distinct entity that assigns additional capabilities to interrelated entities, these are described by the relator's roles. The crux lies in the modelling of these *(relational) roles* which describe the mediation between the ar-

relations

relata

arity

relators

(relational) roles

guments and the relation or relator, respectively. The (meta-)relation between the (categorial) roles of a relation and the corresponding relata is named «plays» which is subsumed by the ontological basic relation «dependent-on» because roles depend on their player and on complementary roles, viz the totality of roles involved in the relator, cf. [Herre et al. 2006, p33f].

As relators can be seen as instantiations of (categorial) relations, the corresponding relator's roles are instances of a relation's (categorial) roles (a more fine grained view is given in fig. 13.11). Fig. 13.3 summarizes these new aspects in a UML-style diagram which introduces the classical relational view as derivable (the entities marked by "/") from the relator or the relation, resp.; the diagram can be read bivalently as either class or object diagram depending on focussing either relations or relators.

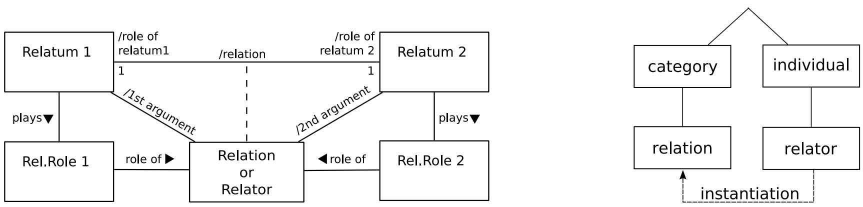


Fig. 13.3: GFO's Relation and Relator

role hierarchy

The problem with roles resides in the simple fact that they are highly dynamic entities, whereas the classical conceptual modelling approach prefers a domain's dissection into more or less static entities. Therefore, roles prefer to be separated from material entities and tend to form a hierarchy of their own. Nevertheless, the connection of the roles's (part-of) hierarchy and the classical material subsumption hierarchy adds additional aspects to the above model.

player universal

As roles restrict the super-type of its player, the above class diagram is extended with an abstract universal named «player universal» which is composed of all types of the objects that can be in the «plays» relation towards this role and which serves as a constraint for the type of the relatum[6].

[6] as will be explicated in sec. 13.3.4, Frank Loebe applies «player univerals» to *objects* that play a role not to a class representing the maximal type of the role-players;

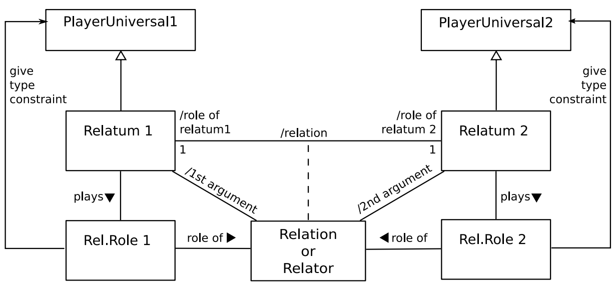


Fig. 13.4: Extending the Diagram with Player Universals

Example 17 : (continued)

In the light of the preceding considerations, $\mathfrak{G}_3$ of fig. 13.2 still lacks the information of the relational roles of the participants of the trust-relation. Further, as one "*not consider[s] the mere collection of the arguments which respect to a single fact [i.e., the entirety of relator and relata as instance of a relation]*" [Herre et al. 2006, p33], relations tend to resemble CG-concepts instead of CG-relations. Moreover, the following demands underpin the choice of relational concepts analogous to post. 9's reification: the demand to model subsumption between relations, e.g., the relation «borrow» as sub-relation of «trust» as well as the composition of relations which is not possible with CGs as only a partial-ordered subsumption hierarchy is admitted [Sowa 2000, p481], and the necessity to annex a relation with additional information, like attributive properties.

Another important subject is the difference between relations that include individuals as the relata and the definition of abstract (universal) relations. As a CG-concept is related by default to the existence of an entity of that concept (see different modes of reference), this distinction does not carry weight in the following CG enhancement. Nevertheless, UML with its distinction between (abstract) class diagrams and communication diagrams which depict the momentary interrelation of objects has to pay attention to this differentiation.

How to properly include these suggestions in the above presented CG framework will be the topic of the next section but one. But first, the above diagram of relators will be transformed to a formally correct UML diagram. This approach seems to deviate from the course of the example; however, the different proposed enhancements to UML introduce new features into the graphical notation that will also play a role when extending CG's graphical repertoire in an analogous way.

13.3.3 A formally revised UML model of GFO's Relator $*$

A rendering of the above sketchy diagrams into standardized UML [OMG 2006] is mainly limited by UML's lack of a differentiation between classes, relators, and roles; especially the latter came into focus over the recent years due to software engineering's (re-)discovery of roles as a basic dynamic modelling pattern [Steimann 2000a]. Before introducing four possible approaches to include roles, some general remarks about UML's extensibility are indispensable.

Some General Remarks to the Extension of UML

Due to UML's embedding into OMG's MOF framework [OMG 2002] which formalizes the meta- as well as the meta-meta-model, the modus operandi of extending UML is embraced in a formal language itself, in this case: again UML, and therefore highly formalized. Further, UML includes several possibilities for enhancements without changing the meta-model, of which additional constraints and the usage of stereotypes are the most prominent. An explicit change in the meta-model is the second way to introduce additional entities or change the usage of the already existing, resp. Third, one can change UML's core semantics from the scratch. Whereas the first approach preserves the validity of old diagrams, the others rely on an explicit translation into the *new* formal language and are therefore called "heavyweight extensions". The next sections will introduce all three approaches towards the inclusion of roles into UML.

ways to extend UML

Introducing «plays»

To model roles as a simple entity type, Jordi Cabot and Ruth Raventós defined the «RoleOf» association which is equal to the above idea behind «plays» as a simple stereotypical enhancement (fig. 13.5) [Cabot & Raventós].

Fig. 13.5: RoleOf Association ([Cabot & Raventós, fig. 1])

Consequently, roles are modelled as entity types, i.e., as classes, whose separation from the type of the entity that bears this role (which are also represented by classes) falls into the responsibility of the modeller. The «RoleOf» association models the attributes that a role adopts if it is played by an associated role-player, e.g., the borrower-role adopts the library-id of the role-bearer's basic type of «being a person».

Cabot & Raventós way of modelling roles is able to describe social roles[7] but not relational roles in the general case.

[7] cf. [Loebe 2007] & [Herre et al. 2006, p39];

Guizzardi's Relator Design Pattern

Considering [Giancarlo Guizzardi 2004], [Guizzardi et al. 2002], and [Guizzardi et al. 2004], Giancarlo Guizzardi proposed a UML notation with ontological foundation [Guizzardi 2005]. Again, this extension makes use of stereotypes whereby a formal ontological approach that resembles GFO is integrated. These stereotypes are embedded into the UML meta-model (see fig. 13.8) without changing its basic structure.

Additionally, stereotypes are accompanied by the other two official ways to enhance UML: tagged values and (OCL) constraints. Further, Guizzardi advocates for the use of design patterns for frequently occurring modelling tasks.

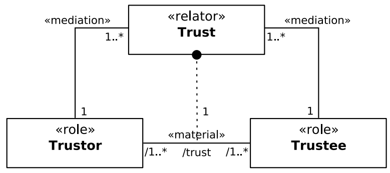


Fig. 13.6: Applying the Relator Design Pattern

The pattern for modelling (material) relations [Guizzardi 2005, p329ff] is depicted in fig. 13.6. A relator mediates between two roles or a role and a kind, i.e., a substance sortal [Guizzardi 2005, p108], e.g., a natural type or concept; the material relation between both can be derived from these mediators (the circle on the end of the dotted association line remarks an enhancement to the traditional usage of association classes).

The connection of roles and the kinds which are subsuming the roles, i.e., the role-bearers, is laid down with the taboo of generalizing kinds with roles and an additional design pattern to model multiple allowed types of a role [Guizzardi 2005, p108, p111, chapter 7]. Guizzardi vehemently disagrees to Steinmann's approach below that infers the strict separation of role and type hierarchies from the prototypical example of disjoint multiple types; the above GFO introduction already included a solution to this problem as player universals. Hence, playing a role simplifies to a simple generalization between a role and a kind classifier. Fig. 13.7 exemplifies the usage of the *disjoint multiple types pattern* as well as the – not further presented – usage of qua-individuals→ [Masolo et al. 2005] which allow to solve some classical role-based paradoxa.

disjoint multiple types pattern

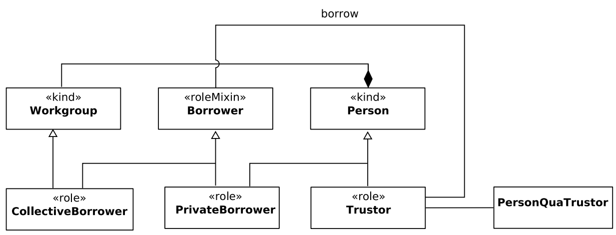


Fig. 13.7: Role-playing as a Generalization and Qua-Individuals

Steimann's Meta-model Enhancement

As already mentioned, [Steimann 2000b] lists the shortcomings of the current UML specification towards roles and mends these by an extension of the UML meta-model (fig. 13.8). Roles are already included in UML as an important feature of communication diagrams (the former collaboration diagrams) but not in the stricter sense of role modelling.

Steimann demands a strict separation of role and type hierarchies, and requires that association-ends connect only roles. Roles are introduced as a new metaclass→ which

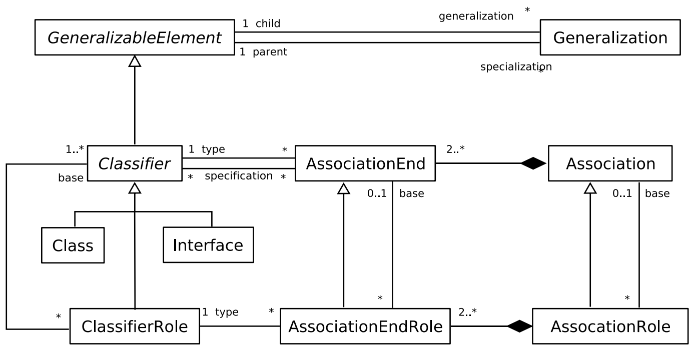


Fig. 13.8: Excerpt of Structural Form of UML Meta-model (due to [Steimann 2000b, fig. 1])

subsumes the classical interface and classifier-role. Therefore, roles are depicted like interfaces either as ○ or as stereotyped classes (resembling the above «role» stereotype) and the «populates» (meta-)relation which relates a class with the roles their instances can play as interface-generalization ┈┈▷. Following [Steimann 2000b], the old style diagrams stay valid and furthermore can easily be translated to the new notion.

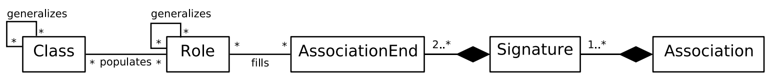


Fig. 13.9: Revised Meta-model ([Steimann 2000b, fig. 3])

This chapter's standard example is given in fig. 13.10. Attributable to the meta-model, both trust relations refer to only one association which represents «trust» with two different signatures of which only the lower one is associated to the role-filler's type.

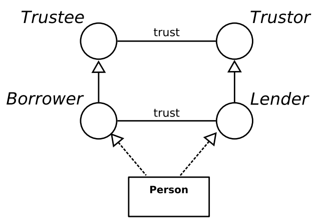


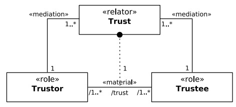
Fig. 13.10: The Basic Example with Roles as Interfaces

A New Core Semantics

As a last resort, the change of the underlying semantics would be the most heavy-weight change possible. On top of the long list of complaints about UML's current semantics, this approach, though leading to a totally different language, can be by no means regarded as unfounded. Based on the above approach, [Steimann & Kühne 2002] proposed a new semantic core which integrates with the standard diagrammatic notion and implicitly includes roles and time as basic entities. Since roles would be represented as interfaces, the resulting diagram of this approach resembles fig. 13.10 with a different underlying semantics.

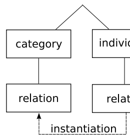
Relations & Relators

The inclusion of relators is straightforward as already seen: towards the relation, a relator can be regarded as an association class of a certain «relator» stereotype which binds a relator to mediate between role types; this was already depicted in fig. 13.6. The relata are introduced via the attached role players by either interface implementation or class specialization. Therefore, the Player Universals coincide with – taking Guizzardi's idea of mixins→ and the presented design pattern – a topmost «roleMixin».



(fig. 13.6)

The contraposition of (categorial) relations and (individual) relators should be depicted by class and objects in UML. Regarding example 17, the formal definition of trust requires a categorial relation, therefore, one would preferably use a class diagram. The concrete prototype can either be depicted via an object diagram (the fameless counterpart of the well-known class diagram), a communication diagram which would include the situations dynamics, or a class diagram enhanced with a way to depict instantiation[8]. The next section will enhance the simple GFO relation model of fig. 13.3 to include the complete, sophisticated GFO-approach.



(fig. 13.3)

[8] cf. the mapping of OWL's instances and classes into UML [Brockmans & Haase 2006], [Schreiber 2002];

13.3.4 An Expert's Review of GFO's relations ∗∗

As elaborated in [Loebe 2003] and [Loebe 2007], GFO's modelling of relations has grown more subtle than the above given original approach. The following diagram and discussion is based on a personal discussion with Frank Loebe and uses an enhanced class diagram style[9]. Instantiation is modelled via a general dependency relation ········➤ tagged with "::" and the instantiating entities are called "individuals"; stereotypes are used to explicate the according categorial type or derived ("/") categorial names which give additional information. For example, the entities instantiating a player universal are often called "players" according to a certain "context".

[9] further, this could serve as an additional example, that scientists prefer sharing their ideas diagrammatically on white- or blackboards;

An important change to the previous considerations is the refinement of the previous definition of «player universal» as the maximal type constraint of a role bearing entity into a class; this step lifts a role-player from the instance level and will further be called «(role) player universal». This class is accompanied by a «natural kind» that constrains the types of the role-bearers. "*Frequently, where player universals are specializing some natural kind, player universals may be considered maximal with respect to that natural kind, i.e., they comprise all objects of that natural kind which actually play their corresponding role*"[10]. Nevertheless, player universals cannot be

[10] personal correspondence with Frank Loebe;

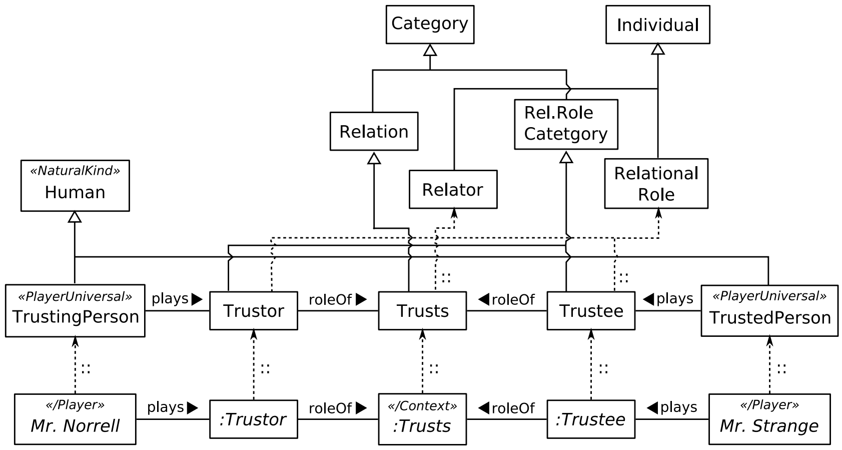


Fig. 13.11:
The Subtleties of
Gfo's Relation Model

constructed as a maximal mixin→ of all a role's bearers, hence, its maximality is only based top-down from the natural kinds.

The prototypical trust relation between two player instances takes place in the lowest row of fig. 13.11: «Mr. Norrell», as the individual entity subsumed under the player universal, plays the individual role (depicted as object) that instantiates the role category «Trustor». Further, this role individual is in the «roleOf» association towards the relator individual that instantiates the relation «Trusts». The important feature is the differentiation between instantiation and generalization: «Trusts» is a relation (generalization) that is simultaneously an instance of the (meta-)category relator. Another important distinction lies between the similarly named associations of the instance- and the categorial level. For example, the «plays» relations between instances has another semantic grounding than the categorial relations, nevertheless they depend on each other[11].

[11] cf. the analogous usage of two different «partOf» relations, one between instances and the other between universals, both named "part of" in [Herre et al. 2006];

Hence, a general definition of a special relation conforming to example 17 has to give a *role base*, i.e., a relation («Trusts») with its relational roles («Trustor», «Trustee») and the natural category which the according player universal specializes (both are «Persons»). The differentiation between role and class types is hidden behind the demand of a natural kind subsuming the player universal as opposed to relational roles.

role base

13.4 An extended Cg Approach

Resuming the task of modelling the example domain, the new insights regarding the abstract understanding of the domain's underlying conceptualization are to be included into the modelling itself. Nevertheless, not all suggested features can be expressed with the Cg paradigm; this will lead to a proposal to extend the framework.

13.4.1 From UML to CG? ∗

Regarding the formalized diagrammatic notion of section 13.3.3, an intuitive step would be the translation of the class diagram into a conceptual graph; this is possible as both notions derive from the common ancestor of ER diagrams.

The utilization of conceptual graphs for the verification of UML class diagrams, as proposed [Loiseau et al. 2005], depends on an automatic translation of UML's basic entities: classes, associations (relations), and generalization (subsumption). These are interpreted quite naturally as CG-concepts and special association classes as well as a CG-relation that represents generalization. Regarding associations, a UML association which is by default binary is then translated into a relation concept which is linked via ternary CG relations to the concepts representing the associated classes and an additional concept that holds the association's attributes (multiplicity, being navigable, being ordered, etc.). This procedure is exemplified in fig. 13.12. Most remarkable is the usage of nested graphs to depict Boolean attributes of UML (cf. CG introduction above: ∃ statements).

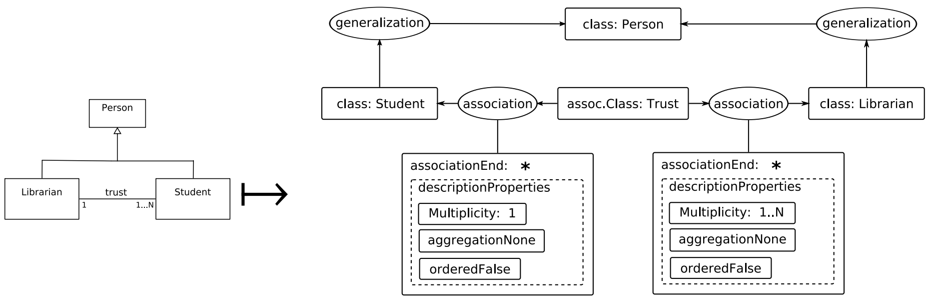


Fig. 13.12: An Example UML to CG Translation

The resulting CGS are a mere rendering of the UML diagram into their notion, i.e., a syntactic translation of one language into the other preserving the features of the original diagram. Nevertheless, these renderings lack the elegance and rigour of a semantic translation but merely contain all the information of the UML diagram[12].

[12] this refers to the principle of information identity as based on the glossary entry on information→ and sect. 2.2 as well as the idea of visual free rides;

13.4.2 Introducing Link-Types

The following approach will try to model [13] relational concepts directly as CGS. As already explicated above, the mixture of relation and object hierarchies, i.e., relation concepts and classical CG concept, must be avoided. Therefore the approach of [Ribière et al. 1993], which was originally intended to enhance the reasoning with CGS's to relationships, gives the desired separation and additionally extends CG with the link formalism of [Fornarino & Pinna 1990] and a new abstraction for link types.[13] This benefit of this approach becomes obvious if one regards the possibility to use

[13] Sowa already introduced links and a link type hierarchy based on Aristotle's analysis of relational links but without rigorous foundation [Sowa 1984];

links between links which allow to deduce new information on a graph due to link-based reasoning.

[Ribière et al. 1993] proceeds as follows: first, there remains only one CG relation →(relation)→ which connects an element of the link type hierarchy with a classic concept; second, both the link type hierarchy and the concept ontology are disjointly combined into a concept lattice whereas both sub-hierarchies only share ⊤ and ⊥. This leads to the situation depicted on the left of fig. 13.13.

link type hierarchy

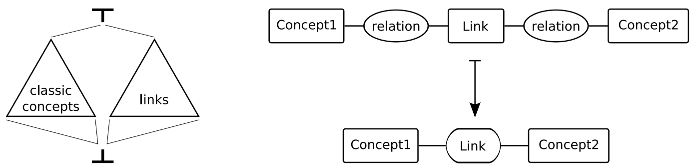


Fig. 13.13: Link Type and Classical Hierarchy; the New Link Node

As there remains only one CG relation, the corresponding nodes will be omitted in the graphical representation. Further, a new style of vertices (LINKTYPE: referent). is introduced to depict link concepts (already applied in fig. 13.13).

Without going into detail, the usage of links between links leads to second order logic[14] because quantification over relations[15] becomes performable. Additionally the possibility to apply either monotonic or non-monotonic reasoning [Ribière et al. 1993] is also beyond first-order. Therefore, relations between relations, e.g., simultaneity, exclusion, or inclusion, can be easily expressed via link type abstraction, i.e., conceptual abstraction for relation concepts.

Hence, the approach of [Ribière et al. 1993] enhances the classical CG framework with conceptualized relations, a strict separation of relation concepts and classical concepts, and, regarding section 12.1, an extension of the corresponding formulae from FOL to second order. These improvements will allow to model the relations of domain more fine grained than the previously introduced notion of CG. Nevertheless, relational roles are still not expressible in this extended framework.

[14] viz [wp:second-order logic] or [Shapiro 2000];

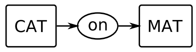
[15] at least according to the authors; whether this is second order or only beyond FOPC but still first order needs to be proven (cf. side note 18 at p39);

relations between relations

13.4.3 Roles and CG Relations

Another requirement mentioned above is the possibility to name the roles of a certain relator. CG relations are already bound to roles in [Sowa 1984, p70f]: "*Conceptual relations specify the role that each percept [or the concept representing this percept, resp.] plays*". Consequently, the graph [Concept1]→(hasRole)→[Concept2] has to be interpreted as "Concept2 plays the role described by hasRole towards Concept1". [Sowa 2000, sect. "Classifying Roles"] and [Sowa 2001b] embed the idea of «has<Rolename>» relations in CG whilst giving a formal foundation.

This application of CG relations overlaps with the approach of utilizing CG relations as conceptual relations itself. Even Sowa did not distinguish these clearly: CG relations are applied in both ways – as roles (see above example) and relations (cf. classical "cat on mat" CG [Sowa 2000, p.477]).



[Sowa 2000, fig. A.1]

Besides the problem of expressing complex relations via simple role-names, this approach has the disadvantage of intermingling roles with the relator which were both assumed to be separated due to the general ontological considerations above.

13.4.4 The Example Domain Revisited: Concept Graphs with Relators

The proposed solution will be a combination of most previously mentioned approaches to model relations. First, relators will be modelled by link types with the appropriate relator taxonomy. Second, the relations of conceptual graphs model the relational roles between a (classical CG) concept and a relator. Third, these roles equally form a hierarchy themselves. Therewith the requirements above are satisfied because role and concept types are separated; furthermore, relators allow reified access to the domain's relations. As the semantic foundation will not be laid down formally in detail, these new graphs will be introduced in the more readable graph theoretic way.

Definition 13.1

concept graphs with relators

Concept graphs with relators are finite, *tri*partite, directed, not necessarily connected multigraphs $\mathfrak{G} = (\mathfrak{C} \cup \mathfrak{L} \cup \mathfrak{R}, E)$.

concept, relator, role, walk

The vertices of the graph are segregated into three types: concepts $\mathfrak{C}$, relators (links) $\mathfrak{L}$, and roles $\mathfrak{R}$. An edge *walk* connects a relator node to either a concept node or a relator node via a single role node[16]. There are no other edges than those participating in a walk, and walks do not cross in roles, i.e., the degree of role vertices is always two.

The special role named *hasRelatum* is the maximal element of a lattice-order $\leq_{\mathfrak{R}}$ on the roles. Further, both concepts and relators form a lattice-order $\leq_{\mathfrak{C}}$ / $\leq_{\mathfrak{L}}$ with maximal element $\top_{\mathfrak{C}}$ / $\top_{\mathfrak{L}}$. These two orders are combined into a single lattice with an additional element $\top$ such that $\top \leq_{\mathfrak{R}/\mathfrak{L}} \top_{\mathfrak{R}/\mathfrak{L}}$ serves as new maximal element whereas the bottom elements coincide $\perp = \perp_{\mathfrak{C}} = \perp_{\mathfrak{L}}$.

[16] without a formal semantic basis of roles, roles between two relators seem dispensable and will be omitted; nevertheless, these entities could describe a new kind of object which could help modelling;

Fig. 13.14 depicts the three defined lattices for concepts, relators, and roles. This trisection allows to apply the classical CG procedures of definition: new concepts and relators can be defined via conceptual abstraction, whereas relational contraction (which is nothing more than a special case of abstraction) is applied to define roles.

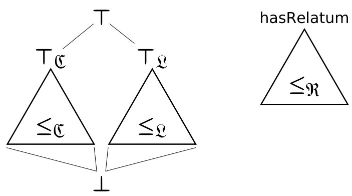


Fig. 13.14: CG with Relators: Three Hierarchies

The maximal element of the (relational) role hierarchy is «hasRelatum» which serves as a default designator for every concept that is attached via a walk to a relator.

Alike the original CGs (cf. sect. 11.2), relators and concepts have a type and referent which can semantically be defined via two vertex labellings as in def. 11.3.

Regarding the semantics of this approach, the only new entities are roles. As with standard CGs, classical concepts and relation concepts are mapped to FCA concepts of $\vec{\mathbb{K}}_0$ and $\vec{\mathbb{K}}_{n>0}$. Therefore, the resulting *partial* semantics which ignores roles, i.e., just assumes the top role «hasRelatum» and interprets it as a graphical feature only, embeds into Dau's FCA approach. Thus a *partial* formal semantics already exists. Advantageous to the mathematizations of Sowa and Dau, concepts and relations share a common lattice analogous to their underlying semantics structures, i.e., formal power contexts, which did not separate both either.

partial semantics

The crux resides in the lack of a formal model of roles, which would require further investigative analysis. Reckoning roles as syntactic sugar only, concept graphs with relators allow to describe real world relations more naturally (compared to current conceptual modelling paradigms) than the standard CG approach which does not allow for the presented subtle differences based on the ontological background of relations.

Additionally, CG framework's notion of concept abstraction has to be extended to relators. This new relator abstraction combines the ideas behind type and relation abstraction as explicated in example 14 (p117). The next example will give a relator abstraction of the trust relation.

relator abstraction with roles

Example 17 : (continued)

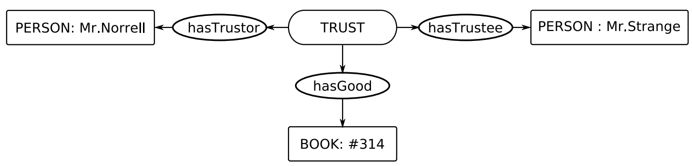


Fig. 13.15: The Example Domain as CG with Relator

Fig. 13.15 shows a possible graph with relators that extends $\mathfrak{G}_3$ of fig. 13.2. Regarding the abstract approach towards trust of section 13.1, the exemplary situation needs a generalized foundation, i.e., a definition of the «Trust» relator which is conform to the above general presentation. This generalization will be given as *relator type abstraction* in fig. 13.16.

relator type abstraction

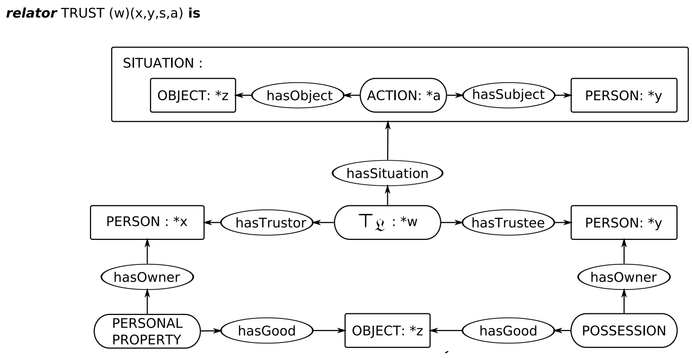


Fig. 13.16: Defining the Trust Relator

The heart of the contraction are two types of coreference: first, w refers to the definiendum but further allows to include subsumption by giving a type more special than $\top_{\mathcal{L}}$; second, the (free) variables x, y, s, and a are the relator's arguments whose roles are given by role vertices and whose player universal is given by the type of the corresponding concept node. Thus, regarding fig. 13.16, the argument x plays the role «hasTrustor» towards the definiendum and must be an object of type «PERSON».

To conclude, the simple «borrow» relation which was mentioned as a prototypical example of a trust relation can be formalized on top of this relator abstraction as in fig. 13.17 whereas the epistemic relators and the (temporal) sequence have to be read "intuitively" without an accompanying CG ontology. Thus, this graph highlights the transition from a situation in which the trustee possesses the object to a situation in which the trustor believes that this object has been returned.

13.4.5 Contrasting the CG and UML Approach $*$

To conclude, the previous sections introduced two models (one with the help of extended UML, the other with conceptual graphs with relators) which both tried to grasp the simple domain of trust. Regarding the requirements that were extracted from the ontological analysis, both notions are able to represent relations or relators as well as the corresponding roles and player universals. The models corroborate the hypothesis of the circulus creativus by explicating the tight, synergetic interplay of the practical usage of a formal language and its foundation.

circulus creativus

Nevertheless, the modelling engineer must choose which paradigm should underlay his modelling and thus respect the subtle difference of pragmatic language handling.

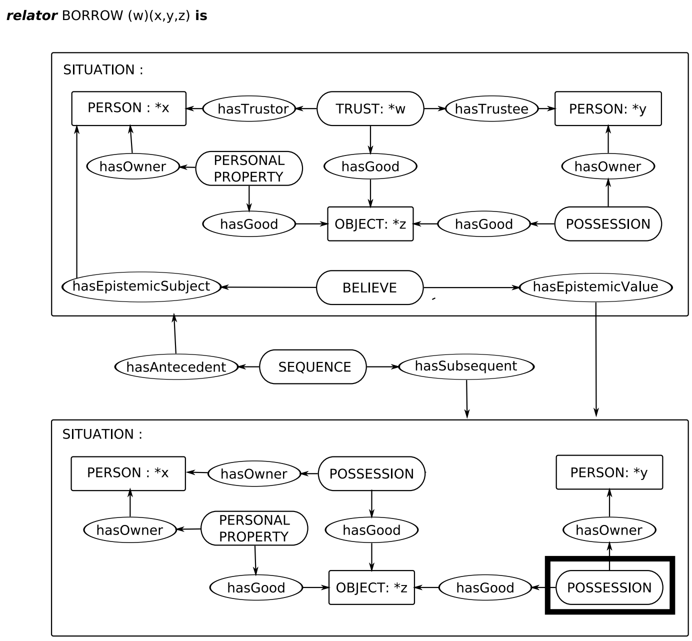


Fig. 13.17: Additionally Defining the Borrow Relator

CGs lack UML's strict separation of classes and objects. More precisely, conceptual graphs do always represent objects of a certain type and no classes[17]. But then, modelling relations beyond the simple associations with UML seems cumbersome, especially as there exist rival approaches to incorporate roles (see sect. 13.3.3) which all lead to profound modifications of the formal language. Contrariwise, the extension of CGs to conceptual graphs with relators leads to a simple and easily readable notion of relators and roles as well as a simple way to introduce new relations by relator abstraction.

[17] whereas the universal quantification over a type $\boxed{\text{PERSON : } *}$ could represent a comprehensive class;

To conclude, the engineer's choice is determined by external factors: UML allows a tight encapsulation into software modelling but requires a more experienced modeller whereas CGs combine an easy way to model relations with a formal semantic bedding.

Another major difference are the ways of extending the language: whereas UML includes[18] ways to enhance the language in itself, from simple stereotypes to a new meta-model, the extension of CGs results in the definition of a new formal language which is based on one of the different possible mathematizations of CG and extends one of the proposed approaches to the semantic foundation. From the meta-modelling point of view, this is surely an advantage of UML.

[18] instead of UML, talking about MOF, the Meta Object Facility, would be more correct;

14 Outlook: Interdependence between CGs and GFO

To draw a first conclusion from the previous chapters, GFO and CG supplement themselves in certain ways. Prospectively, a further exploration of the interplay between both will allow for new inferences in both research areas.

As can be seen in the meta-analysis of sect. 13.3, a core ontology is an usable tool for analyzing the semantic foundation of a formal language or to make this basis explicit. For example, GFO allows for subtle distinctions regarding relations; this procedure helped to reformulate the practical modelling problem of example 17, to explicitly express the underlying problem, and to propose an extension to CG based on these considerations.

semantic foundation of CG

The heart of modelling with CG resides in the underlying ontology (p116). Utilizing GFO would allow both to integrate an ontology with certain proven meta-properties into CG and to explicate GFO-based sentences in a diagrammatic way.

GFO as basic ontology

represent GFO

Further, this facilitates to explicate GFO's Aco itself with CG. Hence, complex definitions and axioms of GFO can be stated in CG instead of FOL which would improve the presentation of GFO axioms in publications (cf. sect. 7.2.5). As already stated in sect. 12.1, a first translation of GFO to conceptual graphs can be simply based on the two renderings of the ontology via FOL (GFO $\stackrel{render}{\longrightarrow}$ FOL $\mapsto$ CG) or with the help of common logic (GFO $\stackrel{render}{\longrightarrow}$ CL $\mapsto$ CGIF).

The previous chapter introduced a new type of nodes into CG that was based on GFO's roles and relators. Several other basic GFO entities could be the starting point for extensions to CG. For example, the implicit relation between situoids and their participating entities as well as the underlying relation to space and time could inspire a situoid-node which would solve the previous problem of the intuitive modelling of temporal relations (cf. fig. 13.17) with the help of GFO's subtle ontological foundation of temporal entities.

extend CG

Consequently, CG could act as a diagrammatic interface to GFO which would combine the formal strictness of GFO with the pictographic usability of CG. First steps in this direction are taken by the Onto-Wiki project which utilizes a CG-like language for knowledge acquisition based on GFO [Backhaus et al. 2007].

15 Resumé of Part III

This part enhanced the previously introduced conceptualizations with comprehensive examples: first, CGs exemplified the introduction of syntax and semantics of a diagrammatic formal language; second, the conceptual modelling of an examplary domain lead to the gap between formal semantics and the semantics as intended by the modeller, which, third, received a foundation by an formal ontological analysis with the help of GFO; consequently, this analysis is fed back to the formal language by an extension proposal that included a better understanding of relations.

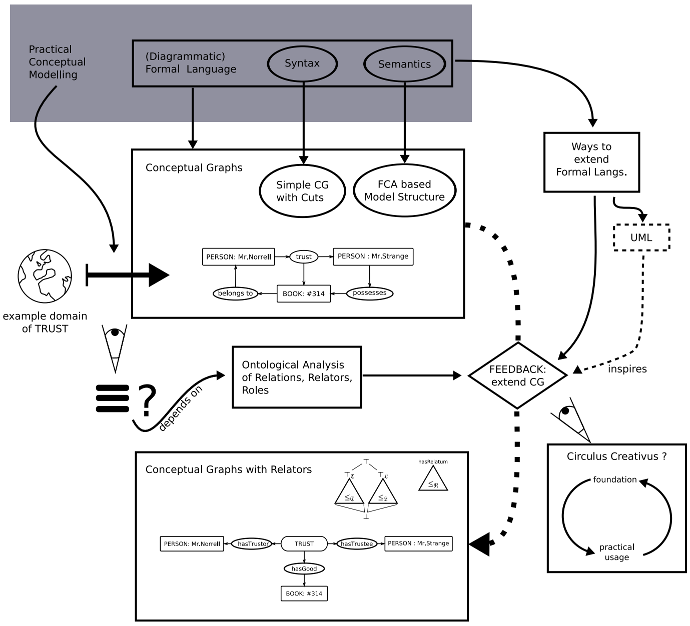


Fig. 15.1: Conceptual Map of Part III

15.1 Notes ∗

Conceptual Graphs

There are different other mathematizations of CGs that – in a basic way – resemble Dau's which was given in detail in the chapters 11 & 12; two newer publications are [Nguyen & Corbett 2006] and [Mugnier & Leclère 2007].

An introduction whose level of detail is between the above given brief entry and the all-embracing [Sowa 1984] can be found in [Sowa 2000, appendix 2]. The proceedings of the annual conceptual structures conference (Iccs) exhibit the most comprehensive overview of all aspects of Cg [Schärfe et al. 2006] , [Dau et al. 2005], [Wolff et al. 2004] , [de Moor et al. 2003], [Priss et al. 2002], [Delugach & Stumme 2001], [Ganter & Mineau 2000], [Tepfenhart & Cyre 1999], [Mugnier & Chein 1998], [Lukose et al. 1997], [Eklund et al. 1996], [Ellis et al. 1995], [Tepfenhart et al. 1994], [Mineau et al. 1993].

An aspect of Cg which was mentioned in example 14 was the usage of predicates to model dependencies in time, e.g., past. These, at a first glance, simple solution lacks a profound semantical basis. Whether, for example, a temporal enhanced Fca [Neouchi et al. 2001] or the usage of contexts (as implemented by the Cg-library CoGITaNT[1]) enhanced with Owl-Time[2] which was discussed recently on the Cg-mailinglist[3], could be part of a possible solution is still an open question.

1 http://cogitant.sourceforge.net/

2 http://www.isi.edu/~pan/OWL-Time.html

3 cf. discussion on Cg-mailinglist starting with http://article.gmane.org/gmane.comp.ai.conceptual-graphs/2085;

Relations and Roles

Section 13.3 already included almost all of the standard literature on roles; again, the work of Frank Loebe should be emphasized who tries to represent these topics between the poles of (Gfo's) ontological rigour and practical implementations [Loebe 2007] [Loebe 2003]. A very coherent and comprehensible introduction is offered by Friedrich Steimann with [Steimann 2000a] and [Steimann 2002].

Yair Wand, Veda C. Storey, and Ron Weber suggested another approach for the foundation of conceptual modelling's relations. They similarly started with basic axioms about the structure of the world, analogous to the postulate of objectivism (post. 9), and tried to approach the semantic meaning of a modelling language's constructs from the perspective of the underlying real world domain. Nevertheless, they lack the 'toolbox' of Gfo which allows for subtle distinctions and a ontological "completeness" per default.

Semantics of Uml

There are different approaches towards the semantic foundation of Uml. [Kent et al. 1999] and [Evans et al. 1999] offer a first (meta-)view onto this topic and also try to enlist all expectations that a semantic foundation should fulfil.

In the following, the most prominent attempts together with their theoretical background will be listed: (a) [Kim & Carrington 2000], [Soon-Kyeong Kim 2000], [Roe et al. 2003] try to apply Object-Z; (b) a category theoretic endeavour can be found in [Smith et al. 2000]; (c) Manfred Broy utilizes his system model [Broy et al. 2006]; (d) the semantics of Uml's dynamic components is focussed by [Jürjens 2002], [Cengarle & Knapp 2004], [Cengarle & Knapp 2005]; and, finally, (e) there is the Gfo point of view [Giancarlo Guizzardi 2004], [Guizzardi et al. 2002].

Part IV

Conclusive Considerations

16 Conclusive Considerations

16.1 A Final Précis

This whole endeavour started with the main research question about the applicability of the pictorial turn to diagrammatic conceptual modelling languages with special regard to their semantic foundation (p2). The course of analysis bypassed various different research areas and examples which all underpin the results that will be summarized in the following.

The central task was to establish a conceptual model of the object of research – the diagrams used in conceptual modelling. Further, the significance of a semantic foundation in the context of these languages had to be fixated; and, as always with formal semantics and its application in practical modelling, the inescapable "semantic gap" had to be faced.

cognition and perception

First of all, the investigation required a formal foundation in a model of cognition and perception as well as semiotics. The starting point was Gärdenfors' idea of conceptual spaces which describe the connection of real-word objects and concepts in an abstract way, i.e., without giving a neuro-biological implementation (sect. 1.3). Nevertheless, the connection of external objects to internal percepts and concepts required a more detailed cognitive model which was introduced in sect. 1.4. This model was designed to explain the "closeness" of pictures to perception with the help of the flow of visual data and the application of a complex self-adapting search algorithm that implemented the matching of percepts to concepts. Furthermore, these results were interrelated to two important other models of cognition: image schemata (sect. 1.7) and artificial perceptions (sect. 1.8).

semiotics

There are different ways to approach the field of semiotics. Chapter 2 combined different possible starting points, e.g., those which are connected with the names like Peirce, Eco, Nørretranders, and Goodman, to name some. This resulted in a comprehensive and elaborate fundament for the introduction of the basic concepts «sign», «communication», «symbol», «semantics», «formal language», and «formal semantics»; on top of these, the famous «semantic gap» and, in an effortless way, the important notions of «conceptual modelling» and «formal ontology» were introduced, notions that would prove to be central to the following analysis. The important feature of this introduction to semiotics was the combination of ideas that originated in theories of different researchers of distinct branches of science (and consequently partially incompatible underlying scientific paradigms). The synopsis of these ideas allowed for many cross connections between different approaches as well as for deducing ways of entering classical fields of discussion from new perspectives. An example would be the introduction of formal ontology as a formalized notion of exformation in the context of an act of communication that takes place in a formal language (viz sect. 2.6).

conceptual model of diagram domain

The next step forward was the conceptual model of the diagram domain. As already suggested by the pictorial turn, the starting point of this investigation was the general notion of pictures.

A conceptual model dissects a domain into objects and their properties; the framework of formal concept analysis (FCA) would allow a description of a FCA-context as the incidence relation between attributes and objects. Hence, FCA is an ideal framework for conceptual analysis but it lacks a standard technique to apply its features to domains that are not given with the help of extremely large relational databases.

This shortcoming led to the design of a simple FCA-based modelling recipe that presents the step-by-step generation of a FCA-context with the help of a simple card-game (viz appendix F). The idea behind the game is the visual exploration of the conceptual search space with the help of visual free rides in a way that is "close to perception". The game itself could be described as the dynamic generation of an abstract diagram of the domain, whereas the outcome of the game can be directly translated to a concept lattice.

In order to apply this simple game, the domain had to be investigated ex ante to derive objects and properties. Both were taken from standard literature and already existing theoretical approaches to the picture or diagram domain. Hence, before delving into practical modelling, sect. 6.1 and 6.2 gave a summary of the most important aspects of already existing conceptualizations and theories about pictorial representation systems. This introduction was based on a meta-model that described the different layers of a picture (sect. 5.3). This meta-model formalized a first naïve approach to pictures and facilitated the interrelation of these different theories.

The FCA modelling failed due to the lack of a rigorous formalization and foundation of both the modelling recipe and the properties which were the basic entities of the game. Nevertheless, two prototypical runs of the game presented the translation of facts that had been extracted from literature to attributes in the resulting FCA lattice.

Further, the outcome of these modelling attempts inspired the following efforts to finally gain a conceptualization of the domain.

A simple axiomatic model of the diagram domain that focussed on diagrammatic formal languages and the corresponding class of abstract logical diagrams allowed to exceed the first modelling attempt and to transfer the results of the analysis of classical linear language to its diagrammatic extension. For example, the idea of the semantic gap still holds in abstract logical diagrams which is the class the most diagrammatic modelling languages belong to; however, the application of diagrams instead of sentences of a linear language establishes new ways of bridging this gap with the help of visual features.

Since the basic problem of the previous FCA modelling was the lack of a formal foundation, one has to revise and formalize the ideas of cognition and semantics with a mathematically rigorous basic model. Consequently, the previously introduced in- or semi-formal models from the Bildwissenschaft literature could be founded on this new model. As this would imply a reformulation of all results of part I, only a first sketch of a categorical model was presented. The choice of category theory as underlying formal language is encouraged by the outstanding results of applying this theory to cognition and formal language semantics.

The last part returned to the underlying research question from a pragmatic starting point. The previous considerations explained the influence of the semantic gap on (diagrammatic) conceptual modelling languages; consequently one such language had to be introduced in more detail.

Chapter 11 introduced conceptual graphs (CG) in a classical way based on the works of Sowa, Dau, and the French School (Mugnier et al.). The following chapter focussed on the problematic field of formal semantic foundation. Again, different already existing approaches were introduced and related; this resulted in an original overview of CG's semantics in connection with the basic ideas of pictorial semantics as presented in part II. Hence, conceptual graphs were introduced as an example of a diagrammatic formal language especially regarding semantic foundation.

A practical conceptual modelling example confronted the semantic gap and the vicious circle between the stable foundation of a language and its dynamic change regarding its practical application. The first attempt to model the simple example domain of trust with conceptual graphs failed. A metaphysical analysis based on the general formal ontology (GFO) revealed that these graphs were incapable to model the domain's peculiarities ab initio: they lack the possibility to express relators and roles. Hence, CG had to be extended, i.e., the underlying formal language had to be changed.

The other important diagrammatic modelling language – Uml– was also shown to be incapable to grasp the example domain. In contrast to Cg, Uml (or Mof, resp.) includes ways to extend the language as part of its formalization such that it includes relators and roles. Hence, the extensions of Uml, both concrete, e.g., how to add role names to associations, and abstract, i.e., how to add new constructions to the formal language itself, could inspire the extension of conceptual graphs.

Finally, sect. 13 proposed the novel, extended notion of conceptual graphs with relators that is – by construction – a simple extension of conceptual graphs which is consistent to the underlying Fca-semantics. Consequently, the vicious circle was shown to be a creative circle in the end.

16.2 Evaluation of the Results

The central question of this work results in an exploration of a large field of research from a new starting point. Hence, there exists no direct answer to the main research question; the underlying investigations resulted in a conceptualization from a new perspective onto the domain of diagrammatic representations. This analyses permitted to provide new insights into the underlying basic concepts.

The picture domain revealed itself as being too extensive, especially regarding the different points of view and layers which are to be taken into consideration. Despite of the failure of trying to grasp the whole domain at a single blow, the proposed simple axiomatic model allows for a comprehensive transfer of the basic results of part I to the diagram domain. As this approach is based on the remains of the previous general Fca approach and consequently the underlying literature research, it is inherently interrelated with other basic theories of the standard literature.

The idea to revise the whole thesis with the help of category theory seems disproportionate and cumbersomely. However, the problems that were faced by the Fca modelling approach, e.g., the implicit interrelations between different standard theories which have not been made explicit before, reveal the need for a more abstract and more foundational approach. Elaborating this basic sketch to an extensive model of the domain would be beyond the scope of this thesis, as it would include an in-depth examination of the mathematical theory of categories.

The third part tries to solve the semantic foundation of diagrammatic modelling languages and the inescapable semantic gap from a different perspective: only the practical application of the creative circle allows to bridge this gap step by step. Nevertheless, the meta-analysis of a concrete discrepancy between a certain formal expression and the intended semantics depends on a sophisticated, subtle, formal ontological framework that allows to make these disagreements explicit. Consequently, the ideas of this meta-analysis feed back into the language; in the case of this thesis: they lead to the proposition of an extension to the framework of conceptual graphs. However, this procedure depends on a proficient usage of the technique of ontological analysis.

Regarding the methodology of this thesis, diagrams are not only mentioned as research objects but took part in the presentation of results. As the success of these diagrams depends on the visual literacy of the readers and the expertise of the writer, on cannot neglect the practical advantages of these pictorial representations, e.g., by giving an overview of different parts of the thesis or by connecting an abstract diagram of an underlying model with the help of icons to the surrounding text.

16.3 Prospective Research

The results of this thesis form a basis for future research. This section will list different possible ways that extend the investigations of the presented undertaking.

Elaborating an Categorical Model

As already emphasized, chapter 8 only outlined a categorical model. This model would start from scratch by proposing a formalized fundament of the introduced concepts, starting from the cognitive model to pictorial properties.

As a result, the ambiguity would be removed and all the smaller models would be integrated in an all-embracing "über-model" which would – by the way – serve as the starting point of a large variety of other research and modelling questions.

To avoid any form of scientific megalomania, the next intermediate research goal could be a simple model of abstractness and visual metaphors which both play an important role in diagrammatic representation. Again, the works of Zippora Arzi-Gonczarowski could serve as a first source of inspiration [Arzi-Gonczarowski 1999] as well as [Lakoff & Johnson 1980], [Lakoff 1987], and [Goguen 2005] or the inclusion of dynamical emergent self-organization phenomena [Atmanspacher & beim Graben 2006].

Include Pictures directly in GFO

GFO does neither include symbol systems nor pictorial representations as first class entities. As pictures and diagrams form an important, large group of objects either directly in application domains, e.g., medical imaging, or as part of the top level, an extension of GFO seems inevitable. There are first attempts to include symbolic entities (and their corresponding semantic entities) into GFO whereas mostly classical sentential objects are focussed.

The contents of parts I and II could serve as a comprehensive overview of the underlying domain. A first, basic approach was introduced in chapter 9 which could be the starting point for a more fine-grained ontology engineering.

CG and GFO: a Source of Mutal Inspiration

Further, as already discussed in chapter 14, conceptual graphs are an ideal representation formalism in the context of GFO, either to represent GFO itself or to utilize GFO as a formal ontological foundation of a diagrammatic conceptual modelling language.

A practical application, e.g., in the fashion of applying a creative feedback circle, would introduce other extensions to standard CG which, for example, would include temporal entities as well as the dynamics of a process-oriented point of view.

Semantic Foundation of Roles regarding Conceptual Graphs with Relators

Chapter 13 enhanced the standard conceptual graphs framework with concept graphs with relators. These graphs were shown to be simple extensions of the standard graphs that allow to model relators and roles. Nevertheless, they lack a formal semantic foundation of their notion of role-nodes and the proposed extension has to be applied to other modelling tasks to prove its suitability for daily use. (Regarding the circulus creativus: this requires further feedback loops.)

Formalizing the Modelling Game

The clever idea to model conceptual modelling as a simple game which corresponds to a diagrammatic notion could serve as a point of contact for a variety of different ideas.

First and foremost, the proposed game needs a formalized notion in the sense of game theory[1]. The most interesting problem that has to be faced is the inclusion of the subjective quality features ("good matching") as these influence the goal condition of the game.

[1] see basic introduction at [sep:game-theory];

A different idea would be to include different players which play concurrently. This could serve as a model for most distributed creative acts, e.g., the collective creation of diagrams[2], wikis, or other forms of knowledge-bases.

[2] viz [Zhang & Norman 1994];

These games resemble the game-theoretic approaches in classical logic based on the ideas of game-theory, e.g., Ehrenfeuch-Fraïssé games, viz [Pietarinen & Sandu 2000] and [sep:logic-games].

Towards Creativity in General

A synoptic view onto the underlying methodology of this thesis reveals the basic pattern[3] of the "creative circle" (p129); examples are: the circle between the genesis of a modelling language and its practical application (cf. ch. 13), or the basic scientific approach to a new domain with the help of gradual approximation by improving models, e.g., with the help of refinement (for example, the different models of cognition of chapter 1), the harmonization of different models (the combination of different semiotic theories in ch. 2), or by choosing a different, more subtle modelling approach (the sketch of a categorical reformulation in sect. 8 that revises and reformulates the results of all previous analyses).

[3] today, "pattern" is a vogue expression in software engineering and modelling; nevertheless, the original work of Christopher Alexander proposes patterns to be the cornerstone of human creativity [Alexander 1964];

On the one hand, this circle is important for theoretical scientific research; on the other hand, it plays a major role for the practical engineer. For example, the "language circle" can be transferred to software engineering and the interplay of the (graphical) representation of code and the engineer's mental model as described in [Storey et al. 1999] and [Storey et al. 2000].[4]

[4] this "language circle" can even be connected to Peirce's approach to logic [Peirce 1983];

Consequently, a formal approach to this circular phenomena would combine a theoretical approach to creativity (cf. [Boden 2002] & [Buchanan 2000]) with the practical application in (diagrammatic modelling) tools. The dynamic aspects of the creative circle underlie the application of diagrammatic conceptual modelling languages whereas the focus pans from the static outcome to the act of creating these diagrams. Regarding [Goodman 1978], this act of "worldmaking" is the heart of modelling.

Appendix

A Diagrams (color)

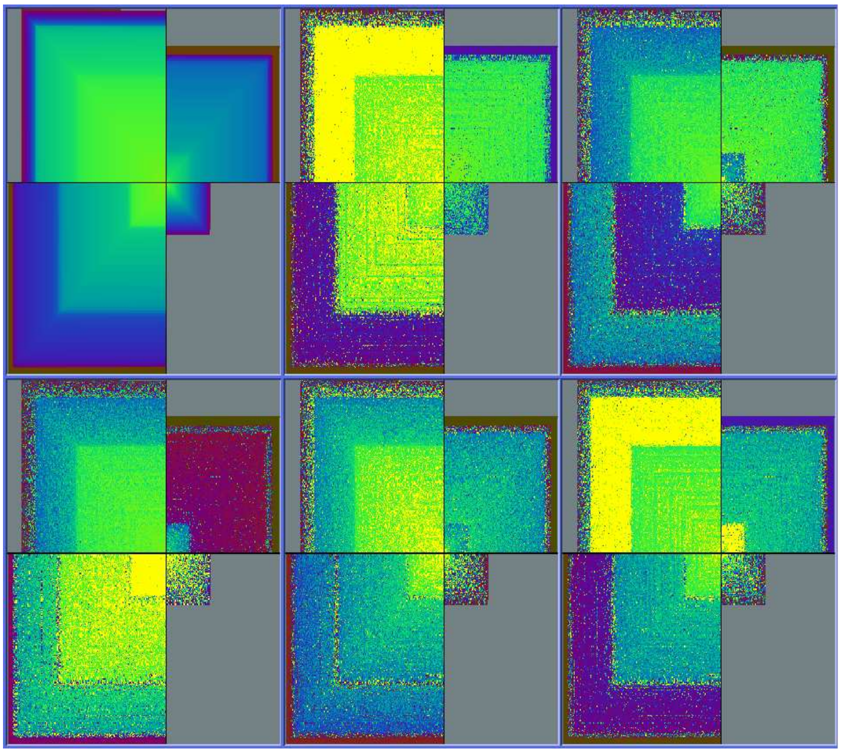


Fig. A.1:
VisDB screenshot:
100000 5 D data itmes
(from [Keim & Kriegel
1994, fig. 8b]

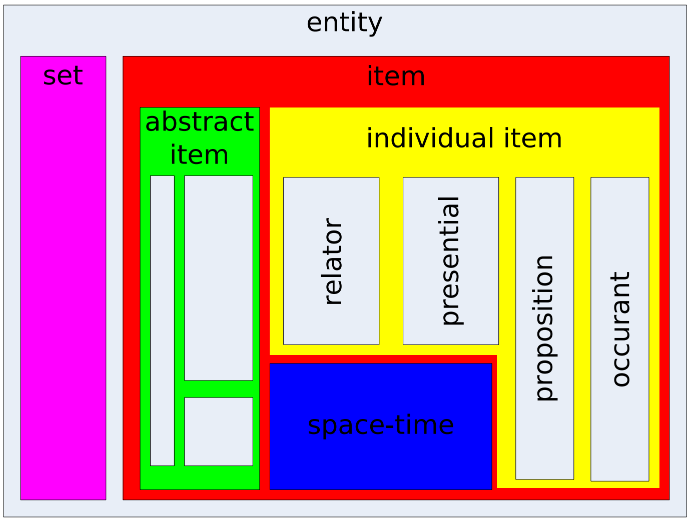


Fig. A.2:
Treemap of GFO Hierarchy

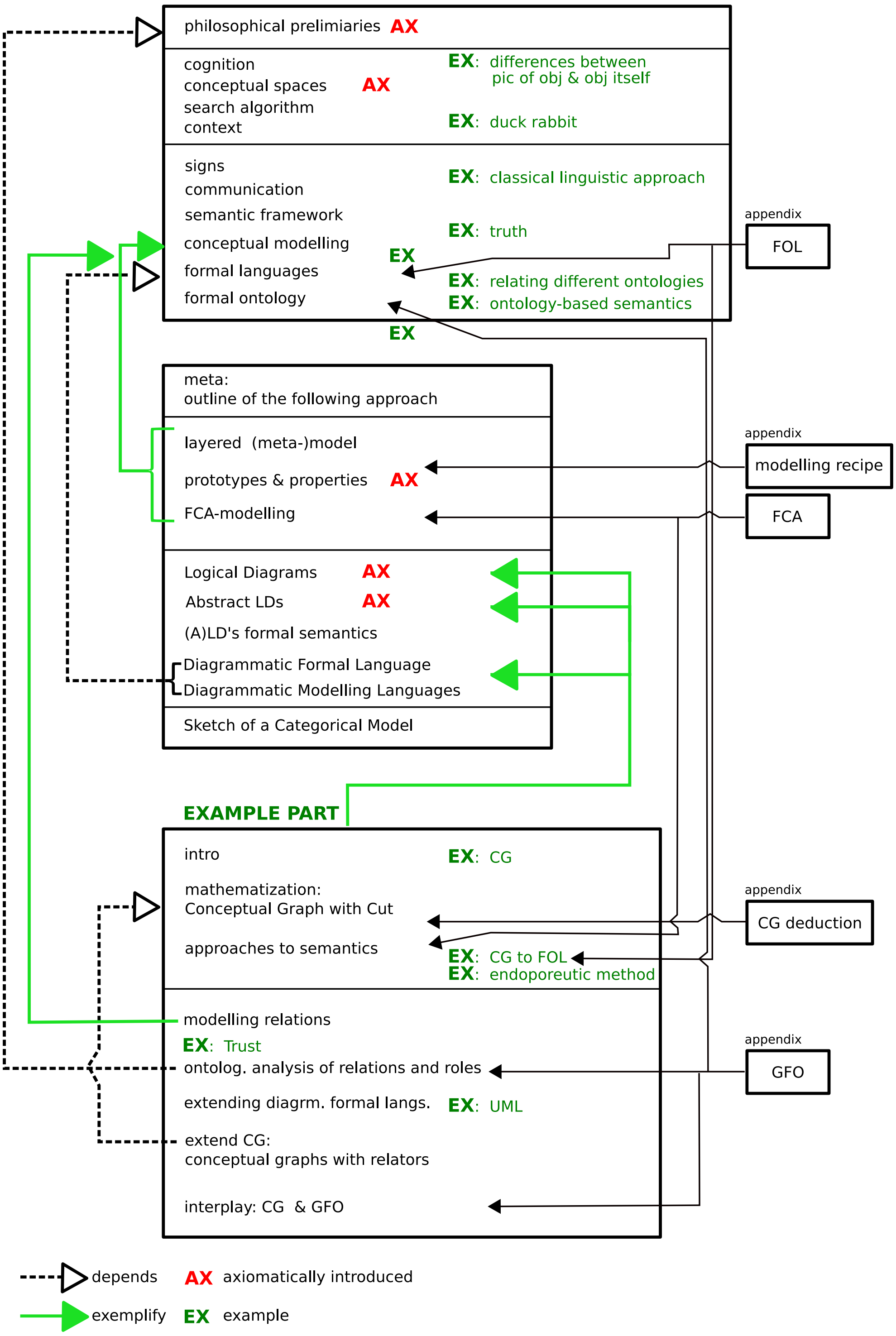


Fig. A.3:
Overview of the Relation of the Relations of this Thesis

B First Order Predicate Calculus

This section will introduce the first order predicate calculus FOPC which is, following sidenote 18 on p39, abbreviated by FOL in this thesis. FOPC serves as both an example of a formal language that is based on the principle of compositionality (cf. p40) and a foundational language for diagrammtic formal languages, e.g., Sowa's foundation for CG in sect. 12.1. A comprehensible introduction into FOPC which is even tracktable for logically untrained readers is given in [Huth & Ryan 2000]; but here, only a short synopsis in the spirit of the introductory examples of [Ebbinghaus & Flum 1995] and [Kuske 2005] will be presented as a comprehensive example of *the* classical formal language.

FOPC / FOL

Signatures and Models

When establishing a formal (algebraic) language, the first step is to restrict the vocabulary to a finite set of signs.

Definition B.1

Signature

> A signature $\sigma = (\mathcal{R}, C, ar)$ is a finite set of relational symbols $\mathcal{R}$ and a finte set of constant symbols C. Every relation has a fixed arity greater than zero expressed by a mapping $ar : \mathcal{R} \rightarrow \mathbb{N}^+$.

Therefore a signature is nothing more than a language's fixed alphabet with an additional classification of the used signs. Conform to section 2.3's definition of formal language semantics, the next step would propose a set-theoretic model structure.

Definition B.2

Model Structure

> A model structure according to a given signature σ, called σ-structure, is a triple $\mathcal{A} = (A, (R^{\mathcal{A}})_{R\in\mathcal{R}}, (c^{\mathcal{A}})_{c\in C})$ with A as finite set called domain or universe. Each constant symbol has a matching element $c^{\mathcal{A}} \in A$ and each relation is presented by tuple elements of the domain according to its arity $R^{\mathcal{A}} \subseteq A^{ar(R)}$.

Syntax

Before framing the syntax of the formal language, some additional semiotic entities must be introduced: (a) a set of variables $V = v_1, v_2, \ldots$ and (b) some logical symbols $=, \wedge, \neg$, and $\exists$.

Terms belong to constant and variable symbols, therefore a term $t \in V \cup C$. Since they are directly related to the model structure, they form the most atomic syntactic units. Consecutively, more complex syntactic entities, *formluae*, can be composed from simpler ones by observing the following rules:

term

formula

(1) if t_1, t_2 are terms then $\phi = (t_1 = t_2)$ is a formula.

(2) $t_1, \ldots t_n$ are terms, $R \in \mathcal{R}$, and $ar(R) = n$ then $\phi = R(t_1 \ldots t_n)$ is a formula

Assuming that ϕ, ψ are formulae and $x \in V$, the following holds:

(3) $\varphi = (\phi \wedge \psi)$ is a formula

(4) $\varphi = \neg\phi$ is a formula

(5) $\varphi = \exists x : \phi$ is a formula

The rules (1) – (5) allow to syntactically construct complex formulae over a given signature. One writes $\phi \in FO[\sigma]$ iff a formula ϕ is derivable by the above rules from the basic terms of σ.

$\phi \in FO[\sigma]$

Semantics

The semantic evaluation of those formulae, regarding a certain underlying σ-structure $\mathcal{A}$, is the second main pillar of compositionality which mirrors the above syntactic construction rules in semantic assignments.

First of all, one defines an *$\mathcal{A}$-evaluation* as a mapping $\alpha : V \cup C \rightarrow A$ such that $\alpha(c) = c^{\mathcal{A}}$ for all $c \in C$. Furthermore, for $x \in V$ and $a \in A$, one defines $\beta = \alpha\left[\frac{a}{x}\right]$ as the evaluation identical to α except that it evaluates the variable x to an element a of the domain. The following rules state the compositional semantics regarding an evaluation α:

$\mathcal{A}$-evaluation

(1) $\mathcal{A} \vDash_\alpha t_1 = t_2$ if and only if $\alpha(t_1) = \alpha(t_2)$

(2) $\mathcal{A} \vDash_\alpha R(t_1, \ldots, t_n)$ iff $(\alpha(t_1), \ldots, \alpha(t_n)) \in (R)^{\mathcal{A}}$

(3) $\mathcal{A} \vDash_\alpha \phi \wedge \psi$ iff $\mathcal{A} \vDash_\alpha \phi$ or $\mathcal{A} \vDash_\alpha \psi$

(4) $\mathcal{A} \models_\alpha \neg\phi$ iff not $\mathcal{A} \models_\alpha \phi$

(5) $\mathcal{A} \models_\alpha \exists x : \phi$ iff there exists a $a \in A$ such that $\mathcal{A} \models_{\alpha[\frac{a}{x}]} \phi$

There are several possiblities to extend the above definition: the $\models$-relation can be extended from $\mathcal{A}$ to a set of formulae, thus restricting the semantics to a sub-structure; further, one has the possiblity to choose between several formal *deduction systems* adding $\vdash$-rules to the above formal language which are conform to the underlying semantics, i.e., $\phi \vdash \psi$ iff $\phi \models \psi$[1], e.g., based on the ideas of Frege and Hilbert or Gentzen.

deduction systems

[1] this is a formal language's (meta-) property of *completeness*; original proof due to [Gödel 1983];

To conlude, the previous paragraphs introduced all parts of a formal language Fopc $= (\sigma, syn, sem, \models, \vdash)$.

Meta-Language

Looking at syntax rule (1) the distinction of meta- and object language becomes obvious. The first "$=$" is part of the meta-language stating the equivalence of formulae whereas the second "$=$" is one of the symbols of object language. The same argumentation works for the semantic mapping of $\wedge$ to an "or"; the latter is to be understood in the common language sense and cannot be founded any further without avoiding the meta-meta-problem.

meta-language

A simple Example

The following section serves as both an example for Fopc formulae and their translation via a compositional semantics.

Assuming $\sigma = \{E\}$ with E being a 2-ary relation-symbol, the model structure $\mathcal{G} = (G, E^{\mathcal{G}})$ is obviously a directed graph$^{\rightarrow}$[2]. The following formula is conform with the syntactic compositioning rules above and must therefore have a semantic interpretation.

$$\phi = \neg\exists x : \exists y : \neg E(y, x) \in FO[E]$$

[2] think of G as set of vertices and the relation E figuratively as arrows connecting two vertices;

This formula can be investigated by the compositionality principle: ϕ is fullfilled in structure if $\psi = \exists x : \exists y : \neg E(y, x)$ is not (4). Due to (5), ψ is accomplished if there exists an element a of the domain to which x can be evaluated to such that $\exists y : \neg E(y, x)$. With the same argument (5), another element b of the domain can be choosen (not necessarily distinct to a) such that $\neg E(y, x)$ is satisfied. This is the case if $E(y, x)$ is not fulfilled, i.e, b is not related via E to a. Summing it all up, ϕ says that there is no such edge such that there is no other edge which is not connected to it, i.e., every edge has a predecessor regarding E.

C Formal Concept Analysis

Formal Concept Analysis (FCA) is a framework for conceptual modelling mainly based on the works of Bernhard Ganter, Rudolf Wille, and Gerd Stumme [Ganter & Wille 1996] [Wille 2000]. The FCA paradigm combines a classical Fregean [+sense, -constr.][1] with lattice theory. This compound allows to apply results of a well-researched mathematical topic to modelling and to depict the concept lattices with easily readable Hasse diagrams[2].

The uniqueness of FCA resides in this lattice-theoretic approach to concepts which was originally applied to data-mining and knowledge representation. Today, FCA plays a prominent role in conceptual modelling by combining the ease of handling (database-) objects that are defined via a list of certain properties to a stringent mathematical model whilst being rooted in a "*formalization of logic by which reasoning is based on 'communicative rationality' in the sense of Pragmatism and Discourse Philosophy*"[Wille 1997, p290].

[1] cf. semantic framework at p32;

[2] Hasse diagrams are order diagrams, they depict the transitive reduction of a partially ordered set [wp:Hasse diagram];

A Short Introduction to FCA

The following simple example will present FCA's two main modes of knowledge representation and serve as an introduction to the understanding of FCA concepts.

Example 18 : A Simple FCA Lattice

> The example domain consists of two well known planets and their moons as well as three simple properties whose correlation is given in the table representation of fig. C.1.

Recapturing Frege's idea to introduce sense ("Sinn")[3] besides the denotation ("Bedeutung") of a term, here a concept consists of its extension, i.e., the objects which are subsumed under the term, and its intension, i.e., the properties shared by the objects of this concept.

[3] see [+sense,-constr.] at p34;

Fig. C.1 introduces a context which lists the example domain's objects and an arbitrary selection of their properties. A formal concept always inhabits a formal context.

	has atmosphere	$\varnothing < 2000km$	is planet
Earth	×		×
Moon	×	×	
Mars	×		×
Daimos		×	
Phobos		×	

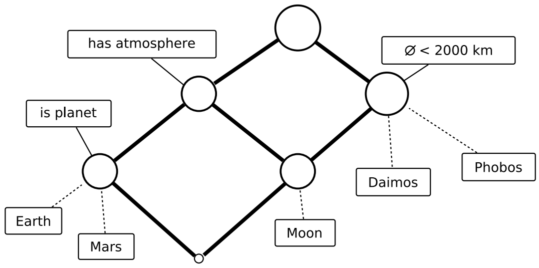


Fig. C.1: FCA Lattice as Table and Hasse Diagram

Definition C.1

context

A *context* is relational structure $\mathbb{K} = (G, M, I)$ with

- a set of objects ("Gegenstände") — G
- set of formal attributes ("Merkmale") — M
- an incidence relation — $I \subseteq G \times M$

Definition C.2

(formal) concept

The tuple (A, B) $(\subseteq G \times M)$ is a *formal concept* :iff
$A = \{g \in G \mid \forall b \in B : \ g\, I\, b\}$ and $B = \{m \in M \mid \forall a \in A : \ a\, I\, m\}$

There are two important projections of concept:
Extension $Ext(A, B) = A$ and *intension* $Int(A, B) = B$.

extension
intension

The definition of an order-relation $\leq$ on all of a context's formal concepts $\mathfrak{B}(G, M, I)$ by $(A, B) \leq (C, D)$:iff $A \subseteq C$ (iff $D \subseteq B$) leads to the complete[4] lattice $\underline{\mathfrak{B}}(\mathbb{K}, \leq)$.

[4] completeness is an important property of lattices, as most result of lattice theory depend on completeness;

Example 18 : (continued)

Back to the previous example, the following FCA-concept describes best the common understanding of «moon»:

$$(\{Moon, Daimos, Phobos\}, \{\varnothing < 2000km\})$$

Its extension contains the moons and no planet; its intension is the property to have a small diameter. On the other hand, it is not possible to find any attributes in this context that would constitute a concept «planet» except the trivial Boolean attribute which depends on an a priori knowledge of planets.[5]

[5] regarding the scientific common sense, the Moon has an atmosphere which is almost negligible, see fact sheet at [NASA 2006];

The strength of FCA remains beyond this simple idea to represent conceptual structures as lattices: the introduction of an implication relation between concepts allows for simple reasoning tasks, and the diagrammatic structure of the lattice diagrams admit to visually explore huge amounts of object-attribute pairs. These diagrams are extensively applied in sect. 6.3 and in the modelling recipe in appendix F.

Modelling Relations

Assuming the convergence of formal concepts and conceptual modelling's concepts (cf. Def. 2.11), the crux of this approach resides in the lack of a possibility to represent relations between concepts. Relations play a central role in conceptual modelling, e.g., example 17 of chapter 13, and cannot be thrown overboard when applying FCA together with other conceptual modelling paradigms like CGS (cf. chapter 13).

The FCA literature proposed two ways to model relations: first, Rudolf Wille suggests to map relations to (special) formal concepts whose intensions consists of a tuple of objects, the relata→ of the relation [Wille 1997]; second, Susanne Prediger extended formal contexts to *relational contexts* by introducing relations as first-class objects, i.e. $\mathbb{K} = ((G, \mathcal{R}), M, I)$ whereas $\mathcal{R} = \bigcup_{k=1}^{n} R_k$; and R_k is the class of all k-ary relations in the domain [Prediger 1998b] [Prediger 1998a] . As the latter approach can blow up the number of objects exponentially, in the following, Wille's proposal which leads from formal contexts to formal power context families will be preferred.[6]

relational contexts

[6] the following definitions are based on [Dau 2003];

Definition C.3

formal power context

A *formal power context* is a family of contexts $\vec{\mathbb{K}} := (\mathbb{K}_0, \mathbb{K}_1, \mathbb{K}_2, \ldots)$ with

$$\mathbb{K}_k := (G_k, M_k, I_k) \text{ such that } G_0 \neq \emptyset \text{ and } G_k \subseteq (G_0)^k \text{ for each } k \in \mathbb{N}$$

Alternatively, one can write $\vec{\mathbb{K}} = (G_k, M_k, I_k)_{k \in \mathbb{N}_0}$.

The elements of G_0 are the objects of the original context definition $\mathbb{K}$; *concepts* are defined over the union of all lattices over contexts of a fixed arity:

concepts

$$c \in \bigcup_{k \in \mathbb{N}_0} \mathfrak{B}(\mathbb{K}_k)$$

and, finally, *relation concepts* are per se those of higher order ($k \in \mathbb{N}^+$):

relation concepts

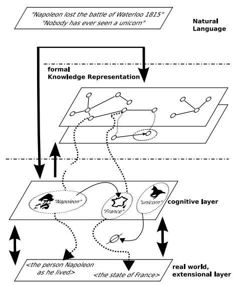
$$r \in \mathfrak{R}_{\vec{\mathbb{K}}} := \bigcup_{k \in \mathbb{N}^+} \mathfrak{B}(\mathbb{K}_k)$$

Formal power contexts lack the simplicity of the standard FCA approach but, nonetheless, allow to use the FCA framework for more complex modelling tasks like the semantic foundation of CGS in section 12.4.

Concepts: Fca vs. Frege vs. Perception

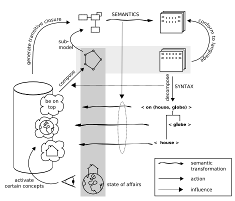
To conclude the short introduction of Fca, the new definition of (formal) concepts will be contrasted to the different definitions of (the concept) «concept» which were already given in the main parts of this thesis.

First, concepts were introduced as units of (subjective) perception (cf. axiom 4, denoted by concept$_{\text{percept}}$ in the following). Second, section 1.3 based these mathematically in the mathematical notion of conceptual space's concepts (concept$_{\text{cspace}}$). Third, entities of the sense layer are often called concept, especially in Frege-biased approaches (cf. p34, concept$_{\text{sense}}$). Fourth, concepts are the basic units of conceptual modelling's dissection of a domain (p42, concept$_{\text{cmodel}}$). Therefore, concepts$_{\text{sense}}$ are a special case of concepts$_{\text{cmodel}}$. Fifth, the previous formal concepts are often formalized with Fca concepts (concept$_{\text{Fca}}$). Analogous to concept$_{\text{cspace}}$, these concepts$_{\text{cmodel}}$ are merely mathematical entities which are denoted by the term "concept" consequently to their interpretation as concepts via a semantic labelling relation. As shown in the example above, concepts$_{\text{Fca}}$ have their origin in the formal semantic foundation of other concepts$_{\text{cmodel}}$, and their separation of a concept's extension and intension resembles the Fregean [+sense] approach save that Carnap's terms are preferred.



(fig. 2.11)

Leaving aside the mathematical notions concept$_{\text{cspace}}$ and concept$_{\text{Fca}}$, one often confuses concepts$_{\text{percept}}$ with concepts$_{\text{cmodel}}$ or concepts$_{\text{sense}}$. The contrast of the perceptive approach with the formal logic approach was already depicted in fig. 2.11 and discussed in example 6 (p42): concepts$_{\text{sense}}$ reside in the intensional layer of Kr as adequate, objective substitutes for real-world entities whereas concepts$_{\text{percept}}$ reside in the cognitive layer either as personal and subjective units of perception, or as inter-personal representation of these which depend on a harmonization process between different persons. Inter-personal concepts form a first bridge towards concepts$_{\text{cmodel}}$ which, anticipating this thesis's world view (cf. philosophical preliminaries p9f), can be grounded on concepts$_{\text{percept}}$ as presented in fig. 2.16. Hence, the step from concepts$_{\text{cmodel}}$ to concepts$_{\text{sense}}$ is equal to the step from [+constr.] to [-constr.] as explicated at p35: concepts$_{\text{sense}}$ assume a internal representation of the whole world (in the terms of fig. 2.11: an isomorphism between the extensional layer and the pre-extensional layer).



(fig. 2.16)

To conclude the discussion about the different notions above, one has to be careful when utilizing «concept» heedlessly. In this thesis, both concept$_{\text{percept}}$ and concept$_{\text{cmodel}}$ play a prominent role and should easily be distinguishable by their context, albeit they can be reduced to each other as presented above.

Another role would be played by Gfo's concepts, but these are still in discussion. Regarding the current state, they would be a hybrid between concept$_{\text{percept}}$ and concept$_{sense}$ which would first demand an inclusion of the psychological stratum into the GFO core to include the underlying cognitive foundation which was necessary due to part I. Possibly, there would be two modularized definitions of concepts regarding the [+constr.] and the [-constr.] approach.

A history of the term concept and an approach that is based on category theory can be found in [Goguen 2005].

D General Formal Ontology

The General Formal Ontology (GFO) is a top-level formal ontology (also known as upper level ontology) and part of the ontological framework IFDAO which is being developed by the Research Group Onto-Med at the University of Leipzig.

Formal ontology is a relatively new branch of computer science although its more general approach could be seen as one of the oldest foundations of science itself: the decomposition of a domain (the world, a concrete application's domain, etc.) into entities and their classification with respect to basic categories. In a nutshell, formal ontology is the study of categories and their interrelations. For example, the question about the basic types or categories of entities in a given domain and the hierarchical[1] structure of these categories, whereas the resulting ontology should be accessible for data processing; following Frank Loebe: Nicola Guarino, Roberto Poli, and Heinrich Herre would not emphasize this dependency on a concrete application in the section of computer science; regarding the previous introduction, their work could be described as transdisciplinary (going beyond the borders of computer science); Poli would describe this difference as "formal ontology versus formalized ontology" [Poli 2003]. A more theoretically based entry to this field is given in part I of this thesis which culminates in a definition of formal ontology (cf. Def. 2.13 on p47) as the integral part of the introduced semantic framework.

formal ontology

[1] in the sense of a a hierarchical polystructure which can be represented by a direct acyclic graph→

Ere the constituents of GFO that play an important role in this thesis, especially Part III's investigation of the semantic foundation of Conceptual Graphs, will be introduced in detail, a short survey of the whole framework is to be presented.

A Primer on GFO

GFO is part of the "Integrated Framework for the Development and Application of Ontologies" (IFDAO) whose predecessor is the GOL-project (General Ontological Language). The IFDAO includes, besides a set of tools for the development and application of ontologies as well as applications relying on basic ontologies, the library of core-ontologies ISFO ("Integrated System of Foundational Ontologies"). GFO is one of these foundational ontologies and, regarding ISFO, the most sophisticated approach [Herre et al. 2006]. The idea behind ISFO is the (meta-) comparison of core ontologies

GFO

IFDAO

ISFO

under the assumption that even core ontologies can be subdivided further into their abstract core (Aco) containing the basic building blocks of the core ontology, e.g., set theory or category theory, an abstract differentiation of top-level categories (ATO), and the basic level entities (BLO) that are the link to the ontology of the underlying application. This trisection was introduced first in [Herre & Loebe 2005] and is applied to GFO in [Herre et al. 2006]; the latter serves as the up-to-date introduction and reference to GFO.

ACO
ATO
BLO

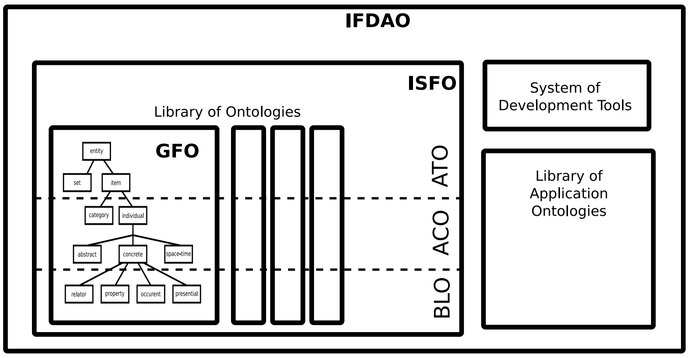


Fig. D.1: Overview of IFDAO

From the viewpoint of ontology engineering, GFO's most distinct feature is its modularity along this three-layered meta-ontological architecture (see above) which allows to combine different sub-ontologies that model a variety of ontological paradigms. The outstanding features of GFO can be summarized as: a "*coherent integration of objects and processes, [... the inclusion of] time and space entities sui generis, [...] an elaborate account of functions and roles [as well as an] openness regarding philosophical positions such as realism, conceptualism, or nominalism by the provision of different kinds of categories as universals, concepts, or symbolic structures*"[Onto-Med].

The GFO Taxonomy

D.2 introduces a taxonomic view onto GFO as presented in [Herre et al. 2006] (A.2 presents the tree diagram as treemap).

To sum up briefly, GFO dissects the most general notion of entities into objects of set-theory, that play an important role in the axiomatization of the residuary taxonomy, and therefore are part of ATO, and items, i.e., the objects focussed in this formal ontology. Aco introduces the classical decomposition into *categories* and *individuals* which refers to the distinction between classes and objects in approaches that are based on object orientation (OO) [Armstrong 2006][2]. At a first glance, these two branches ramify synchronously into *concrete* entities and *abstract* entities as well as

[2] this OO distinction has to be distinguished from the differentiation of universals and individuals [sep:universals-medieval];

categories
individuals

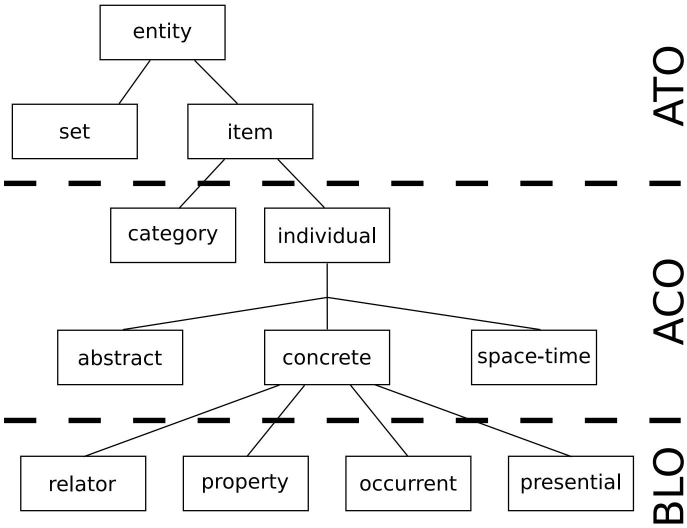


Fig. D.2:
One possible GFO taxonomy

entities of *space and time*, the latter forming a basic pillar of the GFO approach. Again, abstract and concrete entities can be further apportioned by a BLO. A simple approach suggests their disjoint segregation into the following types of entities: (a) *relators* represent relations among items and should therefore not be misplaced with set-theoretic relations; they play an important role in chapter 13; (b) *properties* represent characteristics of things analogous to attributes in OO or slots in a frame-based approach; (c) *occurrents* and *presentials* imitate the opposition of an (abstract/concrete) item's dependency on time: presentials exist wholly at time boundaries whereas occurrents are derived from processes and, thus, perdure in time and cannot be present at time boundaries. For example, the growing up of a child can be described as an occurrent, or more generally: a process; projecting this process onto time-points, e.g., the anniversary of the birthday, results in different presentials of the child which resemble snapshots that are related via the underlying process to the entity which grows up.

concrete vs. abstract vs. space-time

relators

properties

occurrent vs. presential

In the following, only the entities that are playing a prominent role in the main parts of this thesis will be introduced briefly.

Situoids

The most complex kinds of presentials are situations and situoids.

> "*Relations are entities that bind things of the real world together whereas facts are constituted by several related entities [...] together with their relation.*"
>
> [Herre et al. 2006, p33]

relations

facts

> " *A situation is [an aggregation of facts] which can be comprehended as a whole and satisfies certain conditions of unity which are imposed by relations and categories associated with the situation.* "
>
> [Herre et al. 2006, p35]

situation

The affinity of this notion to situation theory was established in [Höhndorf 2005]. Without going into detail, a *process* is a presential which is not present at time-points but can be projected onto these, onto intervals of time, space and consequently to material substrate occupying these regions. A process whose boundaries[3] are "*situations that satisfy certain principles of coherence, comprehensibility, and continuity*"[Herre et al. 2006, p36] is called *situoid.* Situoids are the habitat of occurrents, or, more precisely, each occurrent is embedded in a certain situoid. Therefore, in ontological engineering, situoids can be utilized to represent contexts. This results in the representation of the semantic content of a picture with the help of situoids (cf. 9).

process

[3] in a linear time view: think of its start- and end-points as its boundaries;

situoid

Relators and Relations

(GFO relations are introduced in detail in section 13.3.4 on p138.)

E CG's Diagrammtic Deduction System

This section serves as an example for a formal deduction system for conceptual graphs. The following rules and the example are taken from [Lukose & Kremer 1996, lecture 8]; a complete review regarding an enhanced, sound and complete formal language system is given in [Dau 2003, ch. 5].

Basic Deduction Rules

CG's deduction rules are simple enhancements of EG's inference rules [Dau 2006]. Regarding the nesting of CGs, *evenly and oddly enclosed areas* (the following rules will describe these as "contexts") are areas whose nesting level, when reading the graph outside-in starting from the sheet of assertion, is even or odd; this is equal to the depth in the tree-order of cuts; thus, the sheet of assertion counts as even. even/odd areas

Erasure	In an evenly enclosed context, any graph may be erased, any coreference link from a dominating concept to an evenly enclosed concept may be erased, any referent may be erased, and any type label may be replaced with a supertype.
Insertion	In an oddly enclosed context, any graph may be inserted, a coreference link may be drawn between any two identical concepts, and restriction may be performed on any concept.
Iteration	A copy of any graph $\mathfrak{G}$ may be inserted into the same context in which $\mathfrak{G}$ occurs or into any context dominated by $\mathfrak{G}$. A coreference link may be drawn from any concept of $\mathfrak{G}$to the corresponding concept in the copy of $\mathfrak{G}$. If concepts c_1 and c_2 in some context C are both dominated by a concept c_3 on some line of identity, then a coreference link may be drawn from c_1 to c_2.
Deiteration	Any graph or coreference link whose occurrence could be the result of iteration may be erased. Duplicate conceptual relations may be erased from any graph.

Double Negation	A double negation may be drawn around or removed from any graph in any context.
Coreferent	Two identical, coreferent concepts in the same context may be joined, and the coreference link in-between them may then be erased.
Individuals	If any individual concept c_1 dominates a generic concept c_2 where c_1 and c_2 are coreferent, the $\rho(c_1)$ may be copied to c_2, and the coreference link may be erased.

adapted from [Lukose & Kremer 1996, lect. 8]

These rules have the empty graph, i.e., the empty sheet of assertion $\top$ as the only axiom. In oddly enclosed regions, the given rules of inference add properties: they restrict a concept, add a graph, join new parts to a graph, or add coreference links. On the other side, in evenly enclosed regions, they remove properties: they erase graphs or coreference links, they replace a concept with a more general one, i.e., one which subsumes the original one due to the lattice of ontology.

An Example of Diagrammatic Reasoning

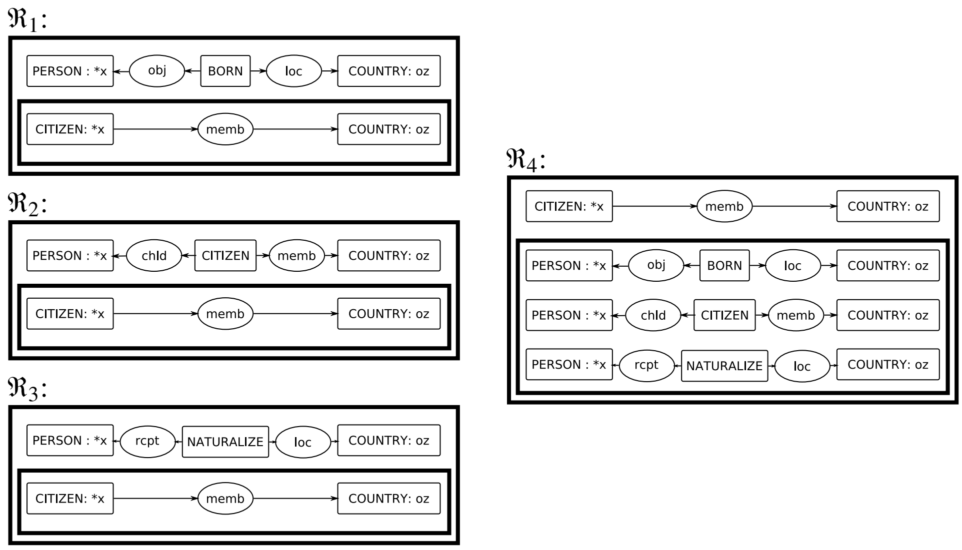


Fig. E.1: An Example Rulebase (due to [Lukose & Kremer 1996])

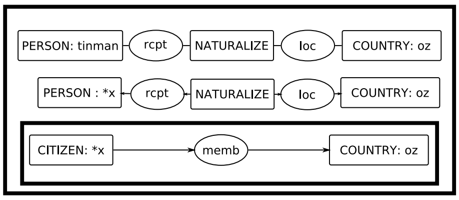
The following example explicates rule-based reasoning on CGs based on the above given inferences. The rules $\mathfrak{R}_1$, $\mathfrak{R}_2$, $\mathfrak{R}_3$, $\mathfrak{R}_4$ are applied to an assertion which is given by an initial graph $\mathfrak{G}$ resulting in a deduced conceptual graph $\mathfrak{G}^{\vdash}$. As already observed in example 16 (p127), simple rules, i.e., material implications, are expressed by a simple doubly nested CG.

The above rules (fig. E.1) describe the membership relation of a person in a country as being equivalent to the property of being either born their, being naturalized, or having the citizenship[1].

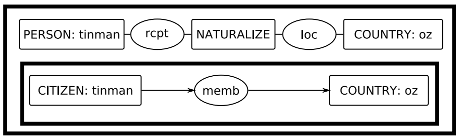
Stating the naturalization of «tinman» in the country «oz»

$\mathfrak{G}$: 

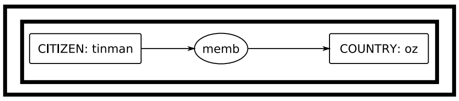
the rule of iteration allows to insert this assertion into $\mathfrak{R}_3$ leading to the following graph



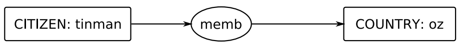
Now, the two oddly enclosed graphs are joined, the general referent ($*$) is replaced by a concrete one which results, due to the coreference $*x$ of $\mathfrak{R}_3$, in a concrete evaluation of the implicated citizen to «tinman». This provides the main result of the rule-based reasoning which is "beautified" by the following steps. Then, similar concepts are joined, i.e., combined to one resulting concept; therefore the two remaining relations between them can be combined too, resulting in



Consecutively, the graph in the odd context can be deleted by deiteration



and the residual double cut can be ignored which leads to the desired result:

$\mathfrak{G}^{\vdash}$: 

[1] as the introduction of CGs did not include disjoint graphs on the same area, their meaning will be introduced as a composition of these graphs' semantic labelling (logical conjunction);

F Recipe: A Game-based Approach to Conceptual Modelling

Original Starting Point: Conceptual Modelling

The goal of conceptual modelling is the generation of a conceptualization (def. 2.10), i.e., a description of certain aspects of a domain in terms of objects, properties, and relations (cf. postulate 9 (objectivism)). As introduced in appendix C, FCA describes both the incidence of attributes in objects as FCA-contexts and ways to model relations (ibid.). Consequently, FCA seems an appropriate candidate to formalize a conceptualization in a conceptual model (def. 2.10).

Basic Problem

The standard techniques of applying FCA to conceptual modelling are based on extracting and deducing FCA-contexts from large relational databases. There is no common way to explore domains from an abstract point of view, i.e., without an underlying empirical database.

Idea

Following hypothesis 7, a modelling engineer first constructs a sketch of the domain. This sketch is often depicted in a graphic notation.

The dynamic generation of this sketch, e.g., by adding and removing entities, properties, and relations, will be formalized below. At heart, the proposed method will interconnect different ways of representing an FCA-context by either a table, a Hasse diagram, a diagrammatic sketch, or a snapshot of a game. The game describes the graphic actions of diagram generation as the moves of a game, hence resulting in a sequence of snapshots of the domain. Without formal proof, these representations are assumed to be equivalent by construction (see fig. F.1). Consequently, the moves of the game step-by-step generate a series of context lattices.

The advantage of using the language of games are the possibility to describe the quality of a formal model, i.e., its "best" matching the domain, as goal of the game and modelling practises to achieve this goal as strategies. The quality of this matching

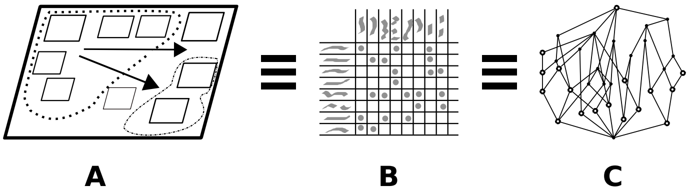
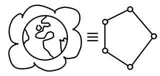


Fig. F.1: Representing the Fca Context by either a snapshot of the Game (A), the Context Table (B), or the corresponding Hasse Diagram (C)

was already embedded in the discussion of knowledge representation and conceptual modelling (p41).



(fig. 2.14)

The crux of conceptual modelling is the subjective quality measure of the outcome, as – regarding the philosophical basics in sect. 1.1 and the discussion of a correspondence theoretic truth (p35ff) – structural isomorphisms to the real world are impossible to define. Hence, the quality measure of being a “better” matching will be put into quotation marks to underline the pragmatic nature.

Reformulating Conceptual Modelling as Game

The main goal of the game is the derivation of a “good” Fca-incidence relation. To achieve this, the following method will propose a step-by-step method which is based upon a visual metaphor that allows to express the searching for a “best” matching conceptualization as a single-player, solitaire-like card-game[1].

The arena or tableau is given by a geometric plane, e.g., a tabletop, that represents the context. The set of objects is represented by a pile of *prototype cards* that bear a prototypical picture of the object; the corresponding attributes are given by properties of these objects that are expressed in a metaphoric way by properties of the diagrammatic plane, e.g., by additional cards representing the center of a radial distance measure, linear scales represented by arrows or coordinate systems, sub-areas of the arena, and colour-markers[2].

[1] a more game-theoretic introduction would introduce a two-player (modeller vs. nature), imperfect information game which possibly runs infinitely but can be stopped at a certain point to get an intermediate result;

prototype cards

[2] this way of marking prototypes allows to get behind the restrictions of building continuous sub-areas in 2 D;

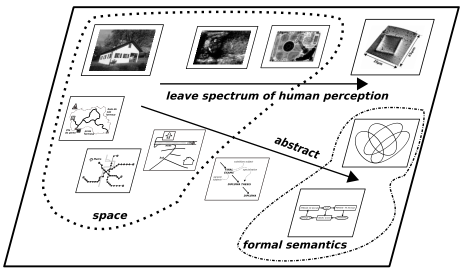


Fig. F.2: Snapshot of a Game: an Intermediate Step of of Sect. 6.3’s Modelling

Rules

The game starts with an empty arena and iteratively adds or deletes prototype cards or properties. The possible moves are:

- adding a prototype card (add an FCA-object):
  first, generate an appropriate card; put this card onto the arena at the position where it "best" matches the already existing properties;

- add a property (add an FCA-attribute):
  after finding a way to express this property, i.e., a symbolic representation by visual variables (p85) of the geometric plane, this property is added to the arena; consecutively, all already played prototypes have to be reordered according to the new set of properties;

- delete either prototype or property
  take away the prototype card or any symbols representing the property; possibly, the remaining prototypes have to be rearranged;

After each move, the set of played prototype cards and the properties applied to them, i.e., the corresponding FCA-objects and attributes, represent an incidence relation or conceptualization, respectively. Fig. F.3 summarizes the results of the move of adding to the underlying concept lattice with the help of the table representation.

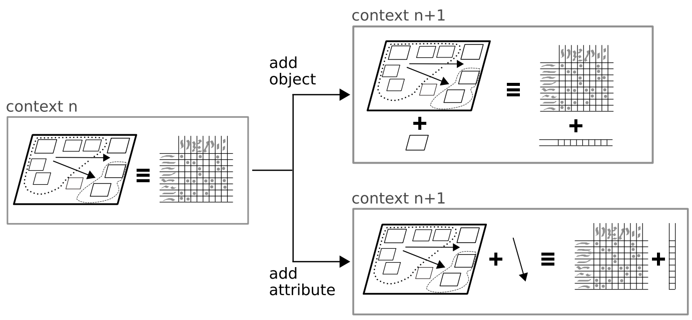


Fig. F.3: Adding an Object or Attribute to the Context is represented by Adding a Prototype or Property in the Game, or by Adding a Row or Column to the FCA Lattice Table

Gameplay

A game that follows the above rules results in a sequence of diagrams starting from the tabula rasa (p70); this corresponds to a series of FCA contexts which emerge from the empty context $(\emptyset,\emptyset,\emptyset) = \mathbb{K}_0 \to \mathbb{K}_1 \to \mathbb{K}_2 \to \ldots$

The goal of the engineer is to obtain a "good" conceptual model. In the language of the game: if the engineer chooses his next move such that it leads to a "better" model, the game is finished if one reaches a fix point, i.e., a local maximum of the matching between specification and domain. The problematic notion in these observations is the quality of this matching. Besides several formal requirements, finding a “good” context depends on the engineers subjective measure. Possibly, the minimal requirements would include the following:

- the model covers most objects with a small number of attributes
- when represented as a Hasse diagram, there are few nodes without an attached objects
- further, it avoids super-nodes, i.e., nodes which collect a large cluster of different objects
- the model includes the common sense understanding of the domain

The latter could be facilitated by the usage of a *goal property*. Goal properties include a previously given classification as (Boolean) predicate, i.e., attribute, «being-of-this-class». For example, the modelling in sect. 6.3 (p90ff) includes a given classification into diagrams, maps, and technical diagrams via three goal-predicates, e.g., «being-a-map». Consequently, the engineer can easily attest that the current snapshot of the game is conform to another already given model if the goal predicates subsume the their prototypes according to the original classification.

goal property

An advantage is the availability of different representations of a single concept lattice. As can be seen in the above itemization, certain qualitative features can be easily verified by eye-catching characteristics of Hasse diagrams (“supernodes”). Hence, playing the game is ideally accompanied by software tools that translate the current snapshot automatically in its corresponding Hasse diagram[3].

[3] for the modelling of part II, the translation to a lattice table was done by hand, and the corresponding diagram was generated by ConExp;

Remarks about the Game

The above modelling game helps to apply Fca to modelling without an underlying large database of object-property pairs which is the classical use case of Fca-based tools. As Fca concepts are close to cognitive concepts (see discussion in appendix C), these concepts propose a simple way to gain a formal founded conceptualization. The simple game above allowed to apply Fca to the domain of diagrammatic representations (cf. sect. 6.3) and to derive at least partial results for the following analyses.

The game as proposed here lacks a formal game-theoretic foundation as well as the proof that a sequence of moves is really able to reach a (local) maximum, i.e., a “best” matching conceptualization, which could be reformulated as a termination condition for the game. Regarding game-theory, the above playing around with cards is far from being a game. Nevertheless, a formal notion of creative processes as modelling games could start with the ideas bind this simple modelling game and even extend it to more than one player.

To conclude, this recipe shows a way to visually explore the conceptual search space. Hence, it can also be regarded as a dynamic generation of a diagram depicting the modelling game. The formulation as a game and the utilization of FCA-based tools to transform the arena via a concept lattice to a Hasse diagram would allow to implement this modelling recipe in a software tool supporting the modelling engineer[4].

[4] for example, elaborated approaches exist in the area of UML's activity diagrams as "exploration games" [Tenzer 2006];

This visual exploration is not far from drawing a corresponding diagram in a step-by-step manner whereas the idea of coding properties is finding an appropriate free ride (cf. sect. 7.2.1). Consequently, the game-based idea could be a formal notion for the creation of diagrams and their underlying graphical morphisms, i.e., a description of the translation between two diagrams with the help of basic operations.

G Glossary

As already explained in sect. 0.4, the following glossary originated in the divers scientific backgrounds of the readers of a preliminary version of this thesis. From a computer science point of view, most of the following entries seem superfluous and – regarding this discipline's mathematical rigour – "inexact".[1] Nevertheless, the multi-disciplinary focus of this work requires to take into account readers that are unfamiliar with the following termini technici.

[1] "*Are you kidding with the definition of linear spaces?*" "*Computer scientists do not need an unmathematical definition of graphs, they know them by heart!*" [some remarks from undisclosed readers];

Commutative Diagram

This is a special kind of mathematical diagram that illustrates the composition of mappings. In a figurative sense, it can be used to show the composition of actions/operations and the resulting compounds. In fig. G.1: a source can be transformed directly to the goal or via the transformation of an encoded version. However, these two ways lead to an equal result — the underlying diagram "commutes".

$$\begin{tikzcd} \text{Source} \arrow[r, "transformation"] \arrow[d, "encode"'] & \text{Goal} \\ \text{Source'} \arrow[r, "transformation'"] & \text{Goal'} \arrow[u, "decode"'] \end{tikzcd}$$

Fig. G.1: Commutative Diagram

In category theory, commutative diagrams play a special role: they form themselves a category, hence they have a direct formal foundation; further, they are used like equations in algebra; and, finally, they allow a diagrammatic mathematical proof technique called *diagram chasing*.

diagram chasing

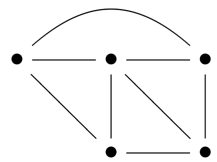
Controlled Natural Language

"*Controlled Natural Languages are subsets of natural languages whose grammars and dictionaries have been restricted to reduce or eliminate both ambiguity and complexity. Traditionally, controlled languages fall into two major categories: those that improve readability for human readers, particularly non-native speakers, and those that improve computational processing of the text.*"

[http://www.ics.mq.edu.au/~rolfs/controlled-natural-languages/]

Graph

In a nutshell, a graph is a set of vertices with edges between them. Normally, a representation of a graph depicts vertices by dots or rectangles and the edges by arcs in-between (see graph representation in fig. G.2 and the prototypical example at p67).



$$G = (\ \{1,2,3,4,5\}\ \{(1,2),(1,3),(1,4),(2,3),(2,4),(2,5),(3,5),(4,5)\}\ )$$

Fig. G.2: Simple Undirected Graph

More mathematically, a (finite) graph is a tuple $G = (V,E)$ such that V is a (finite) set and $E \subseteq V \times V$. A more detailed introduction is given in [Diestel 2005] and [Bollobas 2002].

In this thesis, some graph properties play a prominent role when dealing with conceptual graph's mathematizations (sect. 11.2); these will be introduced briefly:

bipartite
: a graph is bipartite iff one can partition its set of vertices into two whereby each edge only connects a vertex of one partition with one of the other;

undirected
: an edge is undirected iff it connects two vertices without differentiating between them; directed vertices can be depicted as arrows and therefore add additional information to the graph; note: an undirected edge can be translated to two directed edges;
a graph that consists of only (un-)directed edges is called (un-)directed;

complete
: a graph is complete if each pair of vertices is connected by an edge;

connected
: an undirected graph is connected iff one can reach every vertex by a path, i.e., a step-by-step composition of edges, from any other edge;

multigraph
: this graph allows more than one edge between two vertices which are regarded distinct (in contraposition to the mathematical notion above, which would see them as identical and therefore neglect the second one);

In the mathematical field of analysis, a graph (of a function) is the plot of this function which is nt to be confused with the upper definition. graph of a function

Graph homomorphisms are mappings of one graph to another that leave basic structural properties untouched (see also property of homomorphy$^{\rightarrow}$). More formally, the application of such a morphism to a composed graph structure results in the same as the prior mapping of the parts and their later composition. This can be compared to the principle of compositionality (def. 2.9) which is merely a strict homomorphism between syntax and semantics, i.e., an isomorphism. graph homomorphisms

Information

Information can be adequately defined from a stochastic standpoint [Shannon 1948], thus allowing the definition of a measure for information in an exact way.

The starting point is the probability of the occurrence of a probabilistic event $P(event)$. This defines the *self-information* I of this event as self-information

$$I(event) = log_x \left(\frac{1}{P(event)} \right)$$

with x as unit of information, without loss of generality let $x = 2$ (binary units: "bits").

As a simple example, when tossing a coin, the chance for "tail" is $\frac{1}{2}$. When this event occurs, it bears a self-information of $I(tail) = log_2(1/\frac{1}{2}) = log_2 2 = 1$, referring to one bit of information.

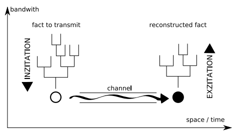
In a nutshell, this leads to a more detailed view of the underlying source of the events in the sense of a probability model. Further, it allows to calculate the source's *information entropy* H, the average self-information over all possible events. information entropy

Assuming n different events numbered from 1 to n, one easily gets:

$$H = \sum_{i=1}^{n} P(event_i) \cdot I(event_i)$$

Recapturing the coin example with its two possible events (head/tail), the entropy is $\frac{1}{2} \cdot 1 + \frac{1}{2} \cdot 1 = 1$, i.e., 1 bit is enough to encode all outcomes of this source (without proof).

At a first glance, this mathematical approach to define information seems strange from a common sense point of view, but in the context of Nørretrander's fig. 2.3



g. 2.3)

(p27) and the introduction of exformation as counterpart, information becomes an important quality of the underlying communication and the transfer of knowledge. Especially the possibility to measure information (see above) distinguishes this approach and allows to use this quality to investigate communication as information transfer.

Linear Space / Vector Space

A vector space is a set of objects (vectors) on which two operations, called (vector) addition and (scalar) multiplication, are defined which satisfy certain natural axioms [wp:Vector Space].

The well-known Euclidean 2-dimensional space is a prominent example of a vector space. Fig. G.3 also shows an important feature of vector spaces: each vector can be generated from a *basis*, i.e., the vectors of the basis (here $\vec{a}$ and $\vec{b}$) are sufficient to describe all other possible vectors of a linear space, i.e., they are representants for the whole space. Furthermore, the Euclidian space allows

basis

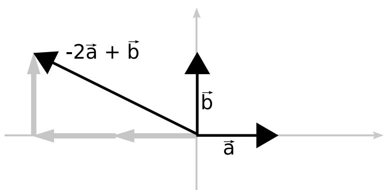


Fig. G.3: Euclidian Vector Space

to use a *metric* to measure the distance of points which correspond to vector's heads (this is defined via a norm of the vector space and is left out of discussion here). For example, the endpoint of vector $\vec{b}$ is at a distance of $2 \cdot length(\vec{a})$ from $-2\vec{a} + b$.

metric

Metaclass

Considering the object-oriented paradigm, i.e., modelling a domain by grouping the common behaviour of its instances by classification [wp:Object-oriented programming], a metaclass is a class who describes classes and their instances [wp:Metaclass].
This extends the ideas of object creation (generalization, etc.) to class generation. Metaclasses are a widely used feature of UML but are only sparsely included in programming languages itself.

Mixin

In object orientation (OO), a mixin is a class that provides a certain functionality which is to be inherited by a subclass [wp:Mixin]. A simple example is the «relator mixin» of the UML-model of Peirce's meaning triangle in fig. 2.1. The

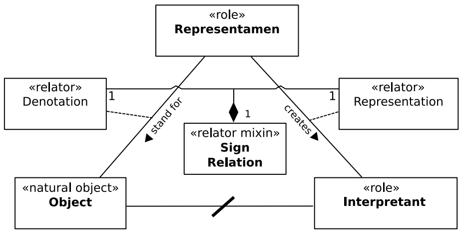


Fig. G.4:
Mixins – Extract of Fig. 2.1

«relator mixin» SignRelation offers the functionality of both sub-«relators»; hence, it represents a kind of functional compositum of these that cannot exists by itself.

Mixins will occur with the ideas of Guizzardi [Guizzardi 2005] which are partially cited in sect. 13.3.3. Many programming languages includes a notion of mixins, others allow to simulate their behaviour of mixins by abstract classes and interfaces [wp:Mixin].

Model Checking

Model checking is an automatic model-based property-verification approach [Huth & Ryan 2000, p. 149]. It compares a specification to a given model and returns whether the specification holds in the model; if not, it tries to produce a counterexample. Model checking originated in the verification of large software systems: as the software's complexity does not allow to effectively (and efficiently) prove the correctness, model checking offers an usable way to check that at least the most vital properties hold.

Qua Individual

Without discussing their existence in reality, qua individuals are a special kind of entity that allow to solve logical dilemmata like the classical "Nixon diamond" [wp:Nixon diamond]: the former president is a Republican and these are usually not pacifist; in addition, he is a Quaker and these are, usually, pacifists.

Qua individuals allow to solve this puzzle: there are two additional individuals, Nixon-qua-Republican and Nixon-qua-Quaker. Hence, Nixon-qua-Republican can order bombings like [wp:Operation menu] whereas Nixon-qua-Quaker can still advocate pacifism. All of Nixon's qua-individuals (Nixon-qua-husband, Nixon-qua-father, etc.) are connected to the single entity «Richard Nixon».

In conceptual modelling, qua individuals are a notion similar to roles, cf. [Masolo et al. 2005][2], and of some importance in certain modelling paradigms.

[2] which additionally contrasts qua-individual to tropes;

Relation

There are two ways to use "relation": first, to describe the ontological connection of ontological entities, and, second, relation in the mathematical sense of set theoretical elements. These two views have to be separated because mathematical relations are often excluded from a formal (core) ontology due to their participation in the meta-ontological language level. The number of related entities is called the *arity* of a relation. The following view onto relations is based GFO [Herre et al. 2006, ch. 10] and is further extended in section 13.3.4 (p138). arity

Ontological Relation

A relation "*bind[s] things of the real world together*"[Herre et al. 2006, p33], these entities are instances of the *relata* of the relation. A relatum expresses the *relational role* of the argument towards the relation. relata relational role

To give a simple example, a word is related to a sentence via a semiotic-part-of-relation. Therefore the relata are word and sentence and the relation forms an abstract category of real world relations. These relations are made of real world instances of words and sentences like this sentence itself. It is sometimes useful to name the relata conform to the role in the (abstract) relation and therefore speak not of words and sentences but of "semiotic parts" and "semiotic wholes".

An all-embracing investigation of relations in the context of semiotics and semantics is conducted by Peirce in [Peirce 1983].

Mathematical Relation

The basic entities of mathematics are sets which are comprise of elements. An n-ary relation over n (different) sets is a n-tuple of elements choosing the first element from the first set and so on. Sets and relations play a crucial role in set-theoretic models (viz appendix B). Albeit the meta-meta-problem, mathematical relations can be investigated with the same instruments as the above ontological relations.

An important property of mathematical relations is *homomorphy*. As the literal translation from Greek discloses, a homomorphous relation, often termed *homomorphism*, conserves certain structural aspects under morphism, i.e., a transformation relation, e.g., a graph homomorphism retains the edges between vertices (see also graph homomorphism$^{\rightarrow}$). homomorphy

Search Space

A way of representing an optimization problem is to describe it as search in a search space. The space represents the domain, i.e., the objects which are to be searched, with an additional function mapping points in the space to real values, i.e., its quality measure to be maximized.

We will take a closer look onto figure G.5's 2D (Euclidian / linear$^{\rightarrow}$) search space with the following added quality measure (represented by an additional 'dimension' as $\mathbb{R}^2 + \mathbb{R}$).

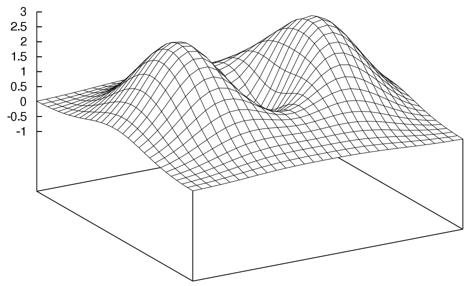


Fig. G.5: Example Search Space

Optimization metaphorically means to find a peak in the landscape whose height represents the quality to maximize. Therefore a search or optimization algorithm has to find the point / vector / element of the plane with the highest peak (global maximum) or at least the highest peak relative to it's neighbours (local maximum).

global / local maxima

One can think of the search algorithm as a hiker's exploration of mountains, launching from a given *starting point* in the landscape and following a trail (a *trajectory*) when searching for a peak.

starting point

trajectory

Stratum

The idea of different ontological levels or *strata* goes back to Nicolai Hartmann who introduced these to describe different levels of the world's complexity [Hartmann 1940]. Roberto Poli advocates their importance in the approaches of formal ontology [Poli 2001] [Poli 1999] but suggests a revised set of levels: the *material* stratum, the *social* stratum, and the *mental* stratum.

These levels describe different (meta-)classes of phenomena and are interdependent, e.g., the social concept of «trust» [3] depends on social entities which themselves interact in a material world. The belief of the trustor in the trustee takes place at the mental level.

[3] as will be detailed in example 17;

For example, the latest version of GFO incorporates the above ideas of strata but mainly focusses on the material level [Herre et al. 2006].

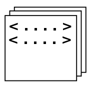
Visual Literacy

"*The term 'visual literacy' was first used by the writer John Debes in 1968 [. . .]. Messarias [. . .] defines visual literacy as the gaining of knowledge and experience about the workings of the visual media coupled with a heightened conscious awareness of those workings. Visual literacy includes the group of skill which enable an individual 'to understand and use visuals for intentionally communicating with others' ([[Ausburn & Ausburn 1978] . . .]). Visual literacy is what is seen with the eye and what is 'seen' with the mind. A visually literate person should be able to read and write visual language. This includes the ability to successfully decode and interpret visual messages and to encode and compose meaningful visual communications*"

[Bamford 2003, p1]

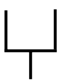
H Iconic Language

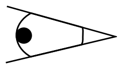
< . . . . > a sentence / expression of a language (different punctuation refers to different sentences), either sentential or pictorial


a collection of expressions, e.g., a sentential text, a logical theory, in conceptual modelling also refers to a representation of a language via all possibly stateable sentences

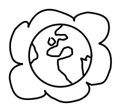

arrow representing the semantic relation (used in both directions)

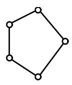

channel transferring without operations (cmp. calculation where this two lines meet, due to [Nørretranders 1997])

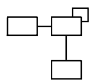

read top-down, represents the simplification of two or more entities into one result or the reverse operation of this calculation (due to [Nørretranders 1997])

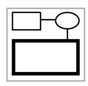

a visual sensor, representing the crossing over the systemic world-agent-border

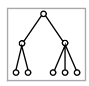

the famous “real world” (see postulate 2)

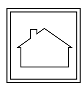

a mental representation of a real world object (post. 4)

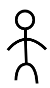

a formal semantic model, e.g., a relational structure

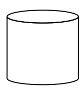

a conceptualization (def. 2.10)

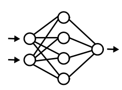

a conceptual graph (ch. 11)


a taxonomy


a pictorial representation of an object, repr. by the frame around it


a cognitive agent


a database or knowledge base


a neural net

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Index

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